INTRODUCING COMPLEX FUNCTIONAL LINK POLYNOMIAL FILTERS

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ABSTRACT

The paper introduces a novel class of complex nonlinear filters, the complex functional link polynomial (CFLiP) filters. These filters present many interesting properties. They are a sub-class of linear-in-the-parameter nonlinear filters. They satisfy all the conditions of Stone-Weirstrass theorem and thus are universal approximators for causal, time-invariant, discrete-time, finite-memory, complex, continuous systems defined on a compact domain. The CFLiP basis functions separate the magnitude and phase of the input signal. Moreover, CFLiP filters include many families of nonlinear filters with orthogonal basis functions. It is shown in the experimental results that they are capable of modeling the nonlinearities of high power amplifiers of telecommunication systems with better accuracy than most of the filters currently used for this purpose.

Index Terms— Nonlinear signal processing, nonlinear filters, complex nonlinear filters, functional link polynomial filters.

1. INTRODUCTION

Functional link polynomial (FLiP) filters have been defined in the real domain [1] and are a sub-class of linear-in-the-parameter nonlinear filters. Their basis functions are polynomials of nonlinear expansions of delayed input samples and follow the constructive rule of the triangular representation of Volterra filters. They satisfy the conditions of the Stone-Weirstrass theorem and thus are universal approximators, for causal, time-invariant, finite-memory, continuous systems defined on a compact domain. They include also many sub-classes of filters with orthogonal polynomials for appropriate stochastic inputs, as the even mirror Fourier nonlinear (EMFN) filters [2], the Legendre [3], the Chebyshev [4], and the Wiener nonlinear filters [5]. The orthogonality of the basis functions allows a fast convergence of adaptive gradient-descent identification algorithms. Moreover, for orthogonal filters it is possible to develop perfect periodic sequences (PPSs) [5–7], which guarantee the orthogonality of the basis functions on a finite interval and allow an efficient identification with the cross-correlation method.

In the complex domain, many filters based on orthogonal polynomials have been proposed mostly for the identification and compensation of radio frequency (RF) high power amplifiers (HPAs) of telecommunication systems [8–15]. It was shown in [8] that the orthogonality of the basis functions greatly improves the condition number of the autocorrelation matrix involved in the least-square (LS) identification of the HPA. The filters proposed in [8–12, 14] are not universal approximators. In fact, their basis function have been specialized only to account for the characteristics of RF HPAs and are composed by the product of an input sample and a polynomial involving modules of delayed input samples. Even introducing cross-terms with the strategy of the generalized memory polynomial filters [16], they do not satisfy the conditions of the Stone-Weirstrass theorem and they are not universal approximators. An exception is given by the Kautz-Volterra filters [17], whose basis functions are orthogonal products of infinite impulse response filtered inputs. The Kautz-Volterra basis functions, however, require the development of an optimal set of poles and will not be considered in the following.

In this paper, we introduce the complex functional link polynomial (CFLiP) filters. These filters are a non-trivial extension to the complex domain of the family of FLiP nonlinear filters and they present many interesting properties. As for the FLiP filters, they belong to the class of linear-in-the-parameter nonlinear filters. They satisfy all the conditions of Stone-Weirstrass theorem for the complex domain and, thus, they are universal approximators for causal, time-invariant, discrete-time, finite-memory, complex, continuous systems defined on a compact domain. They can be based upon orthogonal basis functions, thus guaranteeing good conditioning of the autocorrelation matrix of LS identification, fast convergence of gradient-descent identification algorithms, existence of PPSs that allow the identification of the filter with the cross-correlation method. They also admit many sub-classes of orthogonal filters for specific distributions of the input signal. The CFLiP filters separate the magnitude and phase of the input samples and can be specialized for the identification and compensation of RF HPAs. On one hand, the orthogonal filters of [8, 12] and the generalized memory polynomial filters [16] can be considered as a special case of CFLiP filters. On the other hand, CFLiP filters are able to provide a much more complete estimation of RF HPAs than the orthogonal filters of [8–12, 14] and the generalized memory polynomial filters. In fact, they have much richer phase terms and can better represent the characteristics of HPAs in presence of memory effects, as will be shown in the following.

The rest of the paper is organized as follows. Section 2 introduces the family of CFLiP filters. Section 3 specializes the family for dealing with HPAs. Section 4 provides simulation results about the identification of an HPA model used in the literature. Concluding remarks are given in Section 5.

In what follows, $\mathbb{C}_1$ indicates the unit circle of the complex domain, i.e., $\mathbb{C}_1 = \{z \in \mathbb{C},\ \text{with}\ |z| \leq 1\}$; z^* indicates the conjugate

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of z; $\mathbf{A}^{\mathrm{H}}$ is the Hermitian transpose of $\mathbf{A}$; $\mathbb{N}$ and $\mathbb{Z}$ are the sets of natural and integer numbers, respectively.

2. COMPLEX FUNCTIONAL LINK POLYNOMIAL FILTERS

In this section, we first review the Stone-Weierstrass theorem for the complex domain. Then, we introduce the CFLiP filters and we discuss the families of Orthogonal CFLiP filters.

2.1. The Stone-Weierstrass theorem

Let us consider a causal, time-invariant, finite-memory, complex, continuous nonlinear system with memory of N samples,

$$y(n) = f[x(n), x(n-1), ..., x(n-N+1)], \tag{1}$$

where $x(n) \in \mathbb{C}_1$, $y(n) \in \mathbb{C}$, and f is a continuous function from $\mathbb{C}_1^N$ to $\mathbb{C}$. It is possible to expand the function f in (1) with a series of basis functions $f_i(n) = f_i[x(n), x(n-1), ..., x(n-N+1)]$,

$$y(n) = \sum_{i=0}^{+\infty} c_i f_i(n), \tag{2}$$

with $c_i \in \mathbb{C}$ and $f_i(n)$ a function from $\mathbb{C}_1^N$ to $\mathbb{C}$. Any choice of basis functions $f_i(n)$ defines a different class of filters. Here we want to develop a class of complex nonlinear filters that can arbitrarily well approximate the system on the compact $\mathbb{C}_1$ according to the Stone-Weirstrass (S-W) theorem [18]:

Theorem (S-W theorem)**.** *Suppose $\mathcal{A}$ is a self-adjoint algebra of complex continuous functions on the compact set K, $\mathcal{A}$ separates points on K, and $\mathcal{A}$ vanishes at no point on K, then the uniform closure $\mathcal{B}$ of $\mathcal{A}$ consists of all complex continuous functions on K.*

According to this theorem, every self-adjoint algebra of complex continuous functions on the compact $\mathbb{C}_1^N$ which separates points and vanishes at no point is able to arbitrarily well approximate the continuous function f in (1). A family $\mathcal{A}$ of complex functions is said to be an algebra if $\mathcal{A}$ is closed under addition, multiplication, and scalar multiplication. An algebra $\mathcal{A}$ is self-adjoint if for any complex function $f \in \mathcal{A}$ also the conjugate f^* belongs to $\mathcal{A}$. Note that the S-W theorem for complex functions is identical to that for real functions, apart from the self-adjoint property. In what follows, we develop a class of basis functions $f_i(n)$ that satisfy the conditions of the S-W theorem.

2.2. CFLiP filters

We first develop the set of basis functions for the 1-dimentional case, i.e., for $y(n) = f[x(n)]$ and $N = 1$. We separate module and phase of $x(n)$, considering the polar form $x(n) = r(n)e^{j\phi(n)}$ with $r(n) \in [0, 1]$ and $\phi(n) \in [0, 2\pi]$. Let $g_0[r(n)]$, $g_1[r(n)]$, ..., be a set of real basis functions that satisfy the conditions of the S-W theorem for the continuous real functions defined in $[0, 1]$; $g_0[r(n)]$ is usually assumed equal 1 and $g_i[r(n)]$ can be considered a polynomial of order i. Then, a set of one-dimensional complex basis functions that can arbitrarily well approximate $f[x(n)]$ in $\mathbb{C}_1$ is given

Table 1. Basis functions of order 3 for a CFliP filter with memory N, order $K = 3$, and phase $P = 1$

$\forall\ n_1 = 0, ..., N-1,\ \ n_2 = n_1+1, ..., N-1,\ \ n_3 = n_2+1, ..., N-1$:
$g_3[r(n-n_1)]e^{j\phi(n-n_1)}$
$g_2[r(n-n_1)]e^{j\phi(n-n_1)}g_1[r(n-n_2)]$
$g_2[r(n-n_1)]g_1[r(n-n_2)]e^{j\phi(n-n_2)}$
$g_1[r(n-n_1)]e^{j\phi(n-n_1)}g_2[r(n-n_2)]$
$g_1[r(n-n_1)]g_2[r(n-n_2)]e^{j\phi(n-n_2)}$
$g_2[r(n-n_1)]e^{2j\phi(n-n_1)}g_1[r(n-n_2)]e^{-j\phi(n-n_2)}$
$g_1[r(n-n_1)]e^{-j\phi(n-n_1)}g_2[r(n-n_2)]e^{2j\phi(n-n_2)}$
$g_1[r(n-n_1)]e^{j\phi(n-n_1)}g_1[r(n-n_2)]g_1[r(n-n_3)]$
$g_1[r(n-n_1)]g_1[r(n-n_2)]e^{j\phi(n-n_2)}g_1[r(n-n_3)]$
$g_1[r(n-n_1)]g_1[r(n-n_2)]g_1[r(n-n_3)]e^{j\phi(n-n_3)}$
$g_1[r(n-n_1)]e^{j\phi(n-n_1)}g_1[r(n-n_2)]e^{j\phi(n-n_2)} \cdot g_1[r(n-n_3)]e^{-j\phi(n-n_3)}$
$g_1[r(n-n_1)]e^{j\phi(n-n_1)}g_1[r(n-n_2)]e^{-j\phi(n-n_2)} \cdot g_1[r(n-n_3)]e^{j\phi(n-n_3)}$
$g_1[r(n-n_1)]e^{-j\phi(n-n_1)}g_1[r(n-n_2)]e^{j\phi(n-n_2)} \cdot g_1[r(n-n_3)]e^{j\phi(n-n_3)}$

by $g_k[r(n)]e^{jp\phi(n)}$ for any $k \in \mathbb{N}$ and any p in $\mathbb{Z}$. Indeed, these basis functions and their linear combinations form a self-adjoint algebra that vanishes at no point (for the presence of $g_0 = 1$) and separates points (two separate points differ for module or phase and $g_1[r(n)]$ or $e^{j\phi(n)}$ separates them). We say that the basis function $g_k[r(n)]e^{jp\phi(n)}$ has order k and phase p.

When $N > 1$, we consider the one-dimensional basis functions at time n, $n-1$, ..., $n-N+1$ and then we form all possible products of these basis functions, obtaining the following N-dimensional basis functions $f_i(n)$,

$$g_{k_0}[r(n)]e^{jp_0\phi(n)} \cdot \ldots \cdot g_{k_{N-1}}[r(n-N+1)]e^{jp_{N-1}\phi(n-N+1)} \tag{3}$$

with $k_0, ..., k_{N-1} \in \mathbb{N}$ and $p_0, ..., p_{N-1} \in \mathbb{Z}$. We define the order K of the basis function f_i as the sum of the order of one-dimensional factors, i.e., $K = k_0 + \ldots + k_{N-1}$. We define the phase P of f_i as the sum of the phases of the one-dimensional factors, i.e., $P = p_0 + \ldots + p_{N-1}$. The basis functions in (3) and their linear combinations form a self-adjoint algebra that satisfy all the requirements of the S-W theorem for the approximation of the system in (1).

A CFLiP filter of memory N, order K, and phase P, is the linear combination of the basis functions in (3) with orders from 0 to K and phase P, satisfying the following constraints:

i) All basis functions of order K have phases p_i with $|p_i| \leq k_i$ for $i = 0, ..., N-1$.

ii) All basis functions of order lower than K have phases $p_0, ..., p_{N-1}$ equal to one of the basis function of order K and orders $k_0, ..., k_{N-1}$, lower than or equal to those of this basis function.

The condition i) guarantees the CFLiP filter is composed by a finite number of basis functions, otherwise there would be an infinite number of terms of order K and phase P. The condition ii) guarantees that for any set of phases $p_0, ..., p_{N-1}$ we have all the possible orders $k_0, ..., k_{N-1}$, from $k_i = 0$ till the maximum used value for each i.

For example, Table 1 provides the basis functions of the 3-rd order kernel of a CFLiP filter with memory N, order $K = 3$, and phase $P = 1$.

Eventually, a CFLiP filter of memory N, order K, and phases P_0,...,P_R is the parallel of R filters having memory N, order K, phase P_i, with $i = 1, ..., R$. We will see that a single phase is often sufficient to model the nonlinear systems of some applications. As a matter of fact, in the next sections we will consider $K = 3$ and $P = 1$, thus having CFLiP filters with a number of coefficients equal to $30\binom{N+2}{3} - \frac{39}{2}N^2 - \frac{13}{2}N$. The number of coefficients can be reduced by limiting the diagonal number D_N of the filter, i.e., the maximum time difference between the samples of the cross-products [19].

2.3. Orthogonal CFLiP filters

It is clear that every choice of the real basis functions g_i, defines a different family of CFLiP filters. Some families of orthogonal FLiP filters have been proposed for the approximation of real nonlinear systems. For example, the even mirror Fourier nonlinear (EMFN) filters [2] and the Legendre nonlinear (LN) filters [3] are orthogonal for a white uniform distribution of the input signal in $[-1, +1]$. For any of these filters, we can define the corresponding CFLiP filter, whose basis functions are orthogonal for a white uniform distribution in $\mathbb{C}_1$. For $N = 1$, the basis functions $\widetilde{g}_k(\widetilde{r})$ of these orthogonal filters are defined in the interval $[-1, +1]$, but they can be easily shifted on the interval $[0, 1]$, with a change of variables, considering $g_k(r) = \widetilde{g}_k(2r - 1)$.

In EMFN filters, we have $\widetilde{g}_k(\widetilde{r}) = \cos(k\pi\widetilde{r}/2)$ for k even, $\widetilde{g}_k(\widetilde{r}) = \sin(k\pi\widetilde{r}/2)$ for k odd. In Complex EMFN (CEMFN) filters, with the change of variables we have $g_k(r) = \cos(k\pi r)$.

In LN filters, the functions $\widetilde{g}_k(\widetilde{r})$ are the Legendre polynomials. In Complex LN (CLN) filters, the functions $g_k(r)$ are the Shifted Legendre polynomials [20], given by

$$g_k(r) = (-1)^k \sum_{s=0}^{k} \binom{k}{s} \binom{k+1}{s} (-r)^s. \tag{4}$$

3. MODELLING RF HIGH POWER AMPLIFIERS

HPAs used in modern wireless systems are often operated close to the saturation level to maximize the energy efficiency. Their behavior is therefore highly nonlinear and causes adjacent channel interference and degradation of the achievable bit error rate. While the HPA is typically a memory-less device, the effect of the filters that precede and follow the HPA introduce memory effects that have to be accounted for when modeling the system.

It has been shown in the literature [16] that if the HPA in the RF passband can be modeled as a Volterra filter, then its complex baseband representation is a complex Volterra filter composed only by odd kernels. Each kernels of order $2Q + 1$ is composed by products of $Q + 1$ direct samples and Q conjugate samples. For example, for order 3 we have the following terms,

$$\begin{aligned} &x(n)x(n-n_1)x^*(n-n_2), \\ &x(n)x^*(n-n_1)x(n-n_2), \\ &x^*(n)x(n-n_1)x(n-n_2). \end{aligned} \tag{5}$$

If we replace the input samples in (5) with their polar form, we see that the model is composed by basis functions with phase $P = 1$. As a matter of fact, most of the models proposed in the literature for modeling HPAs are often phase 1 filters, i.e., are composed by basis functions with phase $P = 1$. For example, memory polynomial filters have

$$y(n) = \sum_{k=1}^{K} \sum_{i=0}^{N-1} a_{ki} x(n-i) |x(n-i)|^{k-1}, \tag{6}$$

generalized memory polynomial filters include cross-terms [16],

$$\begin{aligned} y(n) &= \sum_{k=1}^{K} \sum_{i=1}^{N-1} a_{ki} x(n-i) |x(n-i)|^{k-1} \\ &+ \sum_{k=1}^{K} \sum_{i=0}^{N-1} \sum_{l=1}^{D_\mathrm{N}} b_{kil} x(n-i) |x(n-i-l)|^{k} \\ &+ \sum_{k=1}^{K} \sum_{i=0}^{N-1} \sum_{l=1}^{D_\mathrm{N}} c_{kil} x(n-i) |x(n-i+l)|^{k} \end{aligned} \tag{7}$$

where D_N is the diagonal number. In the orthogonal memory polynomial filters [8] we have

$$y(n) = \sum_{k=1}^{K} \sum_{i=0}^{N-1} a_{ki} \psi_k[x(n-i)], \tag{8}$$

where ψ_k are shifted Legendre polynomials (different from those of (4) as explained later) and are a linear combination of the terms in (6). These filters include also even order kernels that have been found important for modeling HPAs [21].

The filters in (6), (7), and (8), can be considered as special cases of the CFLiP filters, but they are not universal approximators because their basis functions do not satisfy the conditions of the S-W theorem. Particularly interesting are the orthogonal memory polynomial filters in (8). It should be notice that the shifted Legendre polynomials ψ_k of [8] are different from those in (4), which follows the standard conventions in mathematics [20]. The ψ_k derives from the orthogonalization of the monomials x, x^2,..., for a uniform distribution of x in $[0, 1]$. The polynomials in (4) derives from the orthogonalization of the monomials 1, x, ... for the same distribution. By choice, the ψ_k do not include a constant term, which is present in g_0 in (4). A family of CFliP filters can be developed also on the basis of the polynomials ψ_k, for $k \geq 1$ and will be named CLN2 in the following. Nevertheless, the CLN2 filters are not universal approximators for complex systems since they do not include a constant term.

It is clear from (5), (6), (7), and (8), that CFLiP filters of phase $P = 1$ could be an interesting candidate for modeling and compensating HPAs. In the next section we will show through simulations that they are capable of improving the modeling accuracy of HPAs compared to the previously mentioned filters. Indeed, CFLiP filters always include cross-terms and they have more complex phase terms than (6), (7), and (8). For example, as shown in Table 1, the CFLiP filter with $K = 3$ and $P = 1$ has the following three phase terms among his basis functions: $e^{j\phi(n-n_1)}$, $e^{2j\phi(n-n_1)}e^{-j\phi(n-n_2)}$, $e^{j\phi(n-n_1)}e^{j\phi(n-n_2)}e^{-j\phi(n-n_3)}$. The first phase term is present in (6), (7), and (8). On the contrary, the other two phase terms, while consistent with the products in (5), are normally not considered in the literature.

Table 2. Identification results for the Wiener-Hamerstein HPA model

Filter	N	K	N_{D}	L	NMSE(dB)	Cond.Num.
CLN	5	3	0	20	-24.7	7.78
CLN	5	3	2	257	-38.5	36.6
CLN	5	3	3	419	-41.6	43.3
CLN	5	3	4	530	-42.3	47.4
CLN2	5	3	0	15	-24.1	2.54
CLN2	5	3	2	103	-37.8	$785.$
CLN2	5	3	3	147	-40.2	$1.12 \cdot 10^3$
CLN2	5	3	4	175	-40.9	$1.32 \cdot 10^3$
CEMFN	5	3	0	20	-24.7	3.38
CEMFN	5	3	2	257	-32.0	10.5
CEMFN	5	3	3	419	-32.4	12.4
CEMFN	5	3	4	530	-32.4	13.6
MP	5	3	0	15	-24.7	$3.30 \cdot 10^3$
MP	5	5	0	25	-24.7	$4.93 \cdot 10^6$
MP	5	7	0	35	-24.7	$6.21 \cdot 10^9$
GMP	5	3	2	39	-24.7	$4.85 \cdot 10^3$
GMP	5	3	3	49	-24.7	$7.42 \cdot 10^3$
GMP	5	3	4	55	-24.7	$9.33 \cdot 10^3$
OP	5	3	0	15	-24.7	2.53
OP	5	5	0	25	-24.7	3.99
OP	5	7	0	35	-24.7	5.48
GOP	5	3	2	39	-24.7	$425.$
GOP	5	3	3	49	-24.7	$719.$
GOP	5	3	4	44	-24.7	$1.14 \cdot 10^3$
Vp1	5	3	0	10	-23.9	30.6
Vp1	5	3	2	47	-29.7	45.5
Vp1	5	3	4	80	-30.4	49.3
Vp1	5	5	0	15	-24.6	916
Vp1	5	5	2	203	-38.1	$1.67 \cdot 10^3$
Vp1	5	5	3	407	-40.8	$1.88 \cdot 10^3$
Vp1	5	5	4	605	-41.4	$2.09 \cdot 10^3$

4. SIMULATION RESULTS

In section we provide some simulation results about the identification of an HPA model. In particular, we consider the Wiener-Hammerstein HPA model used in the experiments of [8]. The model is composed by the cascade of a linear time-invariant (LTI) system $H(z)$, followed by a memory-less nonlinearity, in turn followed by a LTI system $G(z)$, with

$$H(z) = \frac{1}{1.5}\frac{1+0.25z^{-2}}{1+0.4z^{-1}}, \qquad G(z) = \frac{1}{0.52}\frac{1-0.25z^{-1}}{1-0.2z^{-1}}$$

and the memoryless nonlinearity given by the arctan model

$$y(n) = \left(\gamma_1 \tan^{-1}(\zeta_1|z(n)|) + \gamma_2 \tan^{-1}(\zeta_2|z(n)|)\right) e^{j\rho(n)}$$

with $\rho(n)$ the phase of $z(n)$, $\gamma_1 = 8.000\,35 - j4.611\,57$, $\gamma_2 = -3.771\,67 + j12.037\,58$, $\zeta_1 = 2.268\,95$, and $\zeta_2 = 0.8234$. The model has input signal $x(n)$ which is a white uniform noise in $\mathbb{C}_1$ and output signal $d(n)$. The signal-to-noise ration is 80 dB. The model has been identified using different polynomial filters, specifically, the CLN filter, the CLN2 filter, the CEMFN filter, the memory polynomial filter (MP), the generalized memory polynomial filter (GMP), the orthogonal polynomial filter of [8] (OP), and its extension with cross-terms as in generalized memory polynomial filters (GOP), and the complex Volterra filter composed only by phase 1 terms as in (5) (Vp1). All these filters belong to the class of linear-in-the-parameter nonlinear filters and can be expressed with the following input-output relationship,

$$y(n) = [f_1(n), f_2(n), \ldots, f_L(n)] \cdot \mathbf{h} \tag{9}$$

where $f_i(n)$ for $i = 1, ..., L$ are the basis functions at time n, L is the number of basis functions, and $\mathbf{h}$ is the coefficient vector. The coefficients can be estimated with the LS approach:

$$\mathbf{h}_{\mathrm{LS}} = (\mathbf{F}^{\mathrm{H}}\mathbf{F})^{-1}\mathbf{F}^{\mathrm{H}}\mathbf{d}. \tag{10}$$

where $\mathbf{d}$ is a column vector whose n-th term is the HPA model output sample $d(n)$ and $\mathbf{F}$ is the matrix whose n-th row is formed by the basis functions $[f_1(n), f_2(n), \ldots, f_L(n)]$. The filters have been first identified using (10) over 10 000 samples of the input signal and then the normalized mean square error has been measured in following 10 000 samples. Table 2 provides the results of identifications for the different filters and the 2-norm condition number of the matrix $(\mathbf{F}^{\mathrm{H}}\mathbf{F})$. The memory of all filters is $N = 5$, but different orders K and different diagonal numbers D_{N} have been considered for the various filters. As we can see from Table 2, the CLN and CLN2 filters for $D_{\mathrm{N}} \geq 2$ outperform by several dB most of the filters currently considered in the literature (i.e., MP, GMP, OP, and GOP). Similar results have been obtained also for other models of the HPA. The improved modeling accuracy is provided by the richer set of phase terms (i.e., from the use of the phase terms $e^{2j\phi(n-n_1)}e^{-j\phi(n-n_2)}$, $e^{j\phi(n-n_1)}e^{j\phi(n-n_2)}e^{-j\phi(n-n_3)}$) and is obtained at the cost of a larger number of basis functions L. CLN and CLN2 filters provide similar results, with the latter having a lower computational complexity (since they lack the one-dimensional basis functions $g_0(n)$) but a higher condition number. CEMFN filters provide worse identification results because the arctan model has a cusp for $z(n) = 0$ and even mirror functions are inadequate to represent it. Vp1 filters of order 5 provide identification performance similar to CLN and CLN2 filters, at the price of a larger number of coefficients, and have worse condition number.

5. CONCLUSION

CFLiP filters have been introduced in this paper. They belong to the class of linear-in-the-paramenter nonlinear filters and they are universal approximators according to the Stone-Weierstrass theorem. CFLiP filters include many classes of nonlinear filters based on orthogonal polynomials. The orthogonality of the basis functions improves the condition number of the autocorrelation matrix used in LS identification and increases the convergence speed of gradient-descent identification algorithms. Future works will include the non-trivial development of PPSs for CFLiP filters, which guarantee the orthogonality of the basis function on a finite period. Using PPSs as input signals, CFLiP filters can be efficiently estimated with the cross-correlation method, computing the cross-correlation between the basis function and the system output. Moreover, the most relevant basis functions according to some information criterion can also be easily estimated.

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