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# THE WIDELY LINEAR BLOCK QUATERNION LEAST MEAN SQUARE ALGORITHM FOR FAST COMPUTATION IN 3D AUDIO SYSTEMS

*Francesca Ortolani, Danilo Comminiello and Aurelio Uncini*

Dpt. of Information Engineering, Electronics and Telecommunications
Sapienza University of Rome
via Eudossiana, 18,
00184, Rome, Italy

## ABSTRACT

In this paper we propose an algorithm which operates weight adaptation by means of a periodic law and is based on the Widely Linear Quaternion Least Mean Square (WL-QLMS) algorithm. The WL-QLMS successfully handles real-world signals, either proper or improper. However, because of the introduction of full second order statistics into the algorithm, its computational cost is quadruplicated with respect to its precursor QLMS. The proposed Widely Linear Block Quaternion Least Mean Square (WL-BQLMS) algorithm speeds up the execution, thus revealing itself as a good solution in 3D audio signal processing applications, where huge amounts of data are usually treated and signals are typically improper. Simulations exploiting 3D Ambisonic B-Format audio signals provide a report of the WL-BQLMS behavior in comparison with BQLMS.

***Index Terms***— Widely linear, block algorithms, quaternions, hypercomplex, adaptive filters, 3D audio, quaternion Ambisonic representation, B-Format

## 1. INTRODUCTION

In recent years, computational intelligence techniques were extensively used in many audio applications [1, 2]. Today, systems for speech and speaker recognition, speech synthesis and analysis, flanked by sound enhancement solutions, are widespread in either fixed or portable devices. One of the critical aspects in computational intelligence is system modeling. With regards to this point, processing in the *hypercomplex* domains was investigated with the task of handling problems by considering all their physical/mathematical dimensions and any relation among them [3–5]. In particular, in the branch of adaptive filtering, the scientific community widely explored the field of quaternion algebra [6]. After the proposal of a Quaternion Least Mean Square algorithm [7], many other quaternion-valued filters were developed [8–10]. Advantages resulting from the use of quaternions were widely assessed in 3D graphics and avionics, where such a numerical data representation allows the development of filters free from rotation deadlocks [11–14]. In adaptive filtering applications, like weather prediction [15, 16], 3D audio [17] or orientation tracking [18], very interesting results were observed. Because of the peculiar properties of quaternion algebra, it is possible to exploit the correlation among all channels (quaternion dimensions), thus obtaining improved filter performance.

The latest efforts in quaternion-valued adaptive filtering were oriented towards the study of the full second order statistics of the signals [19]. In fact, simulations may fail if the models of the actors participating in the system are inadequate and in adaptive filtering performance may vary notably with the nature of the input signal. Earlier quaternion-valued algorithms did not consider the complementary covariance matrices, making them applicable only to *proper* (or *circular*) signals, i.e. signals featuring a rotation invariant behavior. Unfortunately, most real-world signals are not circular (*non-circular, improper*) [19–21]: examples were found in communications, where imbalance between in-phase and quadrature components gives rise to improper baseband signals [22]. Improper signals, in general, result from multichannel systems having channel gain disparities and correlated components, such as multi-sensor applications [23–26]. In typical array processing applications, if signals (either signal of interest or interference) are improper, it is possible to achieve a high DOA resolution [26]. In [24–26] the *widely linear* model of the signals was exploited. The model takes into account the signal second order statistics by incorporating complementary covariance matrices into quaternion math by means of quaternion involutions.

In [27] a *Widely Linear* Quaternion Least Mean Square (WL-QLMS) filter was presented. However, the shortcoming of WL-QLMS is the computational cost. In fact, the algorithm architecture includes the computation of quaternion-valued convolutions and cross-correlations for all signal involutions. In this work, we propose a solution to this problem. We developed a block version of the WL-QLMS, i.e. it updates the filter weights periodically (Widely Linear Block

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Quaternion Least Mean Square, WL-BQLMS). We applied the algorithm to a 3D audio system, where improper signals are usually encountered. In acoustic applications, we often have to handle long impulse responses, e.g. long reverberation tails, and block algorithms represent a solution in most cases. Moreover, block time-domain algorithms pave the way for the implementation of frequency-domain algorithms [28], which compute transformations over blocks of samples. As known from the properties of the Fourier transform, this mathematical tool allows a fast execution of convolutions and cross-correlations, thus speeding up the processing.

This paper is organized as follows. In Section 2 we recall the important facts of quaternions and augmented (second order) quaternion statistics. In Section 3 we present the WL-BQLMS. The sound field representation in a typical noncircular environment is introduced in Section 4. Finally, results from simulations are given in Section 5.

## 2. INTRODUCTION TO QUATERNION ALGEBRA

### 2.1. Fundamental properties of quaternion math

Quaternions are a geometric algebra and are made up of 4 components: $q = q_0 + q_1\mathbf{i} + q_2\mathbf{j} + q_3\mathbf{k}$.
If $q_0$ is zero, a quaternion is named *pure quaternion*.

The imaginary units, $\mathbf{i} = (1,0,0)$, $\mathbf{j} = (0,1,0)$, $\mathbf{k} = (0,0,1)$, represent an orthonormal basis in $\mathbb{R}^3$ and satisfy the fundamental properties shown below:

$$\mathbf{ij} = \mathbf{i}\times\mathbf{j} = \mathbf{k} \quad \mathbf{jk} = \mathbf{j}\times\mathbf{k} = \mathbf{i} \quad \mathbf{ki} = \mathbf{k}\times\mathbf{i} = \mathbf{j} \tag{1}$$

$$\mathbf{i}^2 = \mathbf{j}^2 = \mathbf{k}^2 = -1. \tag{2}$$

One of the most important properties of quaternion algebra is that quaternion product is non-commutative, i.e. $\mathbf{ij} \neq \mathbf{ji}$ , in fact

$$\mathbf{ij} = -\mathbf{ji} \qquad \mathbf{jk} = -\mathbf{kj} \qquad \mathbf{ki} = -\mathbf{ik}. \tag{3}$$

The sum of quaternions is computed as:

$$\begin{aligned} q \pm p &= (q_0 + q_1\mathbf{i} + q_2\mathbf{j} + q_3\mathbf{k}) \pm (p_0 + p_1\mathbf{i} + p_2\mathbf{j} + p_3\mathbf{k}) \\ &= (q_0 \pm p_0) + (q_1 \pm p_1)\,\mathbf{i} + (q_2 \pm p_2)\,\mathbf{j} + (q_3 \pm p_3)\,\mathbf{k}. \end{aligned} \tag{4}$$

The product between quaternions $q_1$ and $q_2$ is calculated as

$$\begin{aligned} q_1 q_2 &= (a_0 + a_1\mathbf{i} + a_2\mathbf{j} + a_3\mathbf{k})\,(b_0 + b_1\mathbf{i} + b_2\mathbf{j} + b_3\mathbf{k}) \\ &= (a_0b_0 - a_1b_1 - a_2b_2 - a_3b_3) \\ &+ (a_0b_1 + a_1b_0 + a_2b_3 - a_3b_2)\,\mathbf{i} \\ &+ (a_0b_2 - a_1b_3 + a_2b_0 + a_3b_1)\,\mathbf{j} \\ &+ (a_0b_3 + a_1b_2 - a_2b_1 + a_3b_0)\,\mathbf{k}. \end{aligned} \tag{5}$$

The conjugate of a quaternion $q = w + x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$ is defined as $q^* = w - x\mathbf{i} - y\mathbf{j} - z\mathbf{k}$.

The module of a quaternion $q = w + x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$ is defined as $|q| = \sqrt{w^2 + x^2 + y^2 + z^2}$.

### 2.2. Quaternion augmented statistics

In complex numbers, we need both $z = z_a + \mathbf{i}z_b$ and its conjugate $z^* = z_a - \mathbf{i}z_b$ in order to write both its real and complex components as $z_a = \frac{1}{2}(z + z^*)$ and $z_b = \frac{1}{2\mathbf{i}}(z - z^*)$. In the quaternion domain the same task is a little bit harder to achieve. We actually need to define and exploit the quaternion involutions:

$$\begin{aligned} q^{\mathbf{i}} &= -\mathbf{i}q\mathbf{i} = q_a + \mathbf{i}q_b - \mathbf{j}q_c - \mathbf{k}q_d \\ q^{\mathbf{j}} &= -\mathbf{j}q\mathbf{j} = q_a - \mathbf{i}q_b + \mathbf{j}q_c - \mathbf{k}q_d \\ q^{\mathbf{k}} &= -\mathbf{k}q\mathbf{k} = q_a - \mathbf{i}q_b - \mathbf{j}q_c + \mathbf{k}q_d. \end{aligned} \tag{6}$$

Given the definitions above in (6), we can now express each quaternionic component as $q_a = \frac{1}{2}(q + q^*)$, $q_b = \frac{1}{2\mathbf{i}}(q - q^{\mathbf{i}*})$, $q_c = \frac{1}{2\mathbf{j}}(q - q^{\mathbf{j}*})$, $q_d = \frac{1}{2\mathbf{k}}(q - q^{\mathbf{k}*})$ and establish a mapping between quaternion components and involutions.

In order to learn about the *properness* of a signal (Section 2.3), it is helpful to define the complementary covariance matrices:

$$\mathcal{C}_{\mathbf{q}}^{\mathbf{i}} = E\left\{\mathbf{q}\mathbf{q}^{\mathbf{i}H}\right\}, \mathcal{C}_{\mathbf{q}}^{\mathbf{j}} = E\left\{\mathbf{q}\mathbf{q}^{\mathbf{j}H}\right\}, \mathcal{C}_{\mathbf{q}}^{\mathbf{k}} = E\left\{\mathbf{q}\mathbf{q}^{\mathbf{k}H}\right\}. \tag{7}$$

Further details and definitions about second order statistics can be found in [19–21].

### 2.3. On the properness of quaternion-valued signals

Frequently, improper signals are found in real-world applications. One of the goals of research in adaptive filtering is to find a solution of general validity for both proper and improper signals. The *widely linear* model for quaternion-valued signals was introduced to achieve this target [19, 29]. Consequently to simulations, it was demonstrated [19,29] that the WL-QLMS algorithm outperforms the QLMS when the filter input signal is improper.

Properness, or second order circularity, features in those variables having rotation-invariant probability distribution with respect to all six pairs of rotation axes $(\mathbf{1},\mathbf{i})$, $(\mathbf{1},\mathbf{j})$, $(\mathbf{1},\mathbf{k})$, $(\mathbf{i},\mathbf{j})$, $(\mathbf{k},\mathbf{j})$, $(\mathbf{k},\mathbf{i})$. A straightforward way to check the properness of a quaternion random variable $q = q_a + q_b\mathbf{i} + q_c\mathbf{j} + q_d\mathbf{k}$ is to test the following characterizing properties [19]:

1. $E\left\{q_m^2\right\} = \sigma^2, \quad \forall m = a,b,c,d$ (i.e. all four components of $q$ have equal power).

2. $E\left\{q_m q_n\right\} = 0, \quad \forall m,n = a,b,c,d$ and $m \neq n$ (i.e. all four components of $q$ are uncorrelated).

3. $E\left\{qq\right\} = -2E\left\{q_m^2\right\} = -2\sigma^2, \quad \forall m = a,b,c,d$ (i.e. the pseudocovariance matrix does not vanish)

4. $E\left\{|q|^2\right\} = 4E\left\{q_m^2\right\} = 4\sigma^2, \quad \forall m = a,b,c,d$ (i.e. the covariance of a quaternion variable is the sum of the covariances of all components).

A corollary of the above properties is that a Q-proper vector is not correlated with its involutions ($\mathbf{q}^{\mathbf{i}}, \mathbf{q}^{\mathbf{j}}, \mathbf{q}^{\mathbf{k}}$) [30], thus resulting

$$E\left\{\mathbf{q}\mathbf{q}^{\mathbf{i}H}\right\} = E\left\{\mathbf{q}\mathbf{q}^{\mathbf{j}H}\right\} = E\left\{\mathbf{q}\mathbf{q}^{\mathbf{k}H}\right\} = \mathbf{0}. \tag{8}$$

As a consequence, we have that Q-proper signals exhibit vanishing cross-correlation matrices for all components ($\mathbf{q}_a, \mathbf{q}_b, \mathbf{q}_c, \mathbf{q}_d$) and the augmented covariance matrix $E\left\{\mathbf{q}^a\mathbf{q}^{aH}\right\} = 4\sigma^2\mathbf{I}$ is real-valued, positive definite and symmetric ($\mathbf{q}^a = [\mathbf{q}^T\ \mathbf{q}^{\mathbf{i}T}\ \mathbf{q}^{\mathbf{j}T}\ \mathbf{q}^{\mathbf{k}T}]^T$). In addition, all the complementary matrices are real-valued and diagonal. More properties and detailed information about this topic can be found in [19, 30].

## 3. WIDELY LINEAR BLOCK QLMS

The Widely Linear Block Quaternion Least Mean Square algorithm is a block algorithm, i.e. it has a periodic weight update equation. It differs from Block QLMS since it embeds a widely linear model of the signal and it exploits the signal augmented statistics. The algorithm was founded on the revised version of QLMS proposed in [31] and a step-by-step description of it is given just below.

### 3.1. Overview of the WL-BQLMS algorithm

Before running the algorithm, the initial values of all variables and vectors involved need to be chosen. The WL-BQLMS algorithm exploits quaternion involutions, so four sets of filter weights have to be initialized and updated in the process: $\mathbf{w}_{init}$, $\mathbf{h}_{init}$, $\mathbf{u}_{init}$, $\mathbf{v}_{init} \in \mathbb{H}^{M\times 1}$, where $M$ is the filter length. All filter weight vectors ($\mathbf{w}_k, \mathbf{h}_k, \mathbf{u}_k, \mathbf{v}_k$) are defined in a way similar to $\mathbf{w}_k = [w_0[k]\ w_1[k]\ \ldots\ w_{M-1}[k]]^{\mathrm{T}} \in \mathbb{H}^{M\times 1}$.

After the initialization is complete, the following operations are executed at each new input block *k*. Given the input block defined as $\mathbf{x}_k = [x[kL]\ x[kL-1]\ \ldots\ x[kL-L+1]]^{\mathrm{T}}$, its involutions ($\mathbf{x}^{\mathbf{i}}$, $\mathbf{x}^{\mathbf{j}}$, $\mathbf{x}^{\mathbf{k}}$) have to be derived. We denote the filter length and the block length with $M$ and $L$, respectively.

We can now compute the filter output by convoluting each weight vector with its corresponding input involution:

$$\begin{aligned}
y_w[k] &= \mathbf{w}_k^T\mathbf{x}_k = \sum_{l=0}^{M-1} w_k[l]\, x[kL+i-l] \\
y_h[k] &= \mathbf{h}_k^T\mathbf{x}^{\mathbf{i}}{}_k = \sum_{l=0}^{M-1} h_k[l]\, x^{\mathbf{i}}[kL+i-l] \\
y_u[k] &= \mathbf{u}_k^T\mathbf{x}^{\mathbf{j}}{}_k = \sum_{l=0}^{M-1} u_k[l]\, x^{\mathbf{j}}[kL+i-l] \\
y_v[k] &= \mathbf{v}_k^T\mathbf{x}^{\mathbf{k}}{}_k = \sum_{l=0}^{M-1} v_k[l]\, x^{\mathbf{k}}[kL+i-l]
\end{aligned} \tag{9}$$

and summing all four contributions:

$$y[k] = y_w[k] + y_h[k] + y_u[k] + y_v[k]. \tag{10}$$

The filter error at block $k$ is defined as the difference of the desired signal $d[k]$ and the filter output $y[k]$:

$$e[k] = d[k] - y[k]. \tag{11}$$

We can finally update the filter weights:

$$\begin{aligned}
\mathbf{w}_{k+1} &= \mathbf{w}_k + 2\mu \sum_{i=0}^{L-1} e[kL+i]\, \mathbf{x}^*_{kL+i} \\
\mathbf{h}_{k+1} &= \mathbf{h}_k + 2\mu \sum_{i=0}^{L-1} e[kL+i]\, \mathbf{x}^{\mathbf{i}*}_{kL+i} \\
\mathbf{u}_{k+1} &= \mathbf{u}_k + 2\mu \sum_{i=0}^{L-1} e[kL+i]\, \mathbf{x}^{\mathbf{j}*}_{kL+i} \\
\mathbf{v}_{k+1} &= \mathbf{v}_k + 2\mu \sum_{i=0}^{L-1} e[kL+i]\, \mathbf{x}^{\mathbf{k}*}_{kL+i}
\end{aligned} \tag{12}$$

where the step size $\mu$ includes the average term of the block algorithm ($\mu = \mu_B/L$).

### 3.2. Computational Cost

Our goal in implementing the WL-BQLMS was to achieve efficiency from the computational point of view in comparison with WL-QLMS. The critical processing paths are the computation of the filter output and the cross-correlation in the update equation. In WL-BQLMS the number of algorithm iterations is simply divided by the block length $L$ with respect to WL-QLMS. Definitively, we have the computational cost ratio between WL-BQLMS and WL-QLMS equal to $1/L$:

## 4. 3D IMPROPER SOUND FIELDS

As introduced in Section 1, the widely linear model suits those systems having some disparities in the dynamics of its channels and correlated signal components. The 3D audio recording/processing technique called Ambisonics belongs to such a category of systems. This technique employs a coincident array of microphones. According to the particular format used in Ambisonics (see A-Format, B-Format [32]), and ambisonic *order* (a definition is given later on), microphones are oriented towards determined directions (with respect to a reference point) and each have a specific polar pattern. In particular, the so-called ambisonic *1st order B-Format* is configured as in Fig. 1. In such a layout we have one omnidirectional microphone and three figure-of-eight microphones. The capsules are coincident and orthogonal to one another. Because of such a layout (geometric placement with respect to the sound source and different types of polar pattern for each capsule), disparities of the signal levels in the channels are likely to occur.

A detailed explanation about how this 3D audio technique works is not in the scope of this paper. However, it is surely useful to know, as an example, how to get the B-Format signals into a quaternionic representation.

Ambisonics decomposes the sound field into spherical harmonics. In a mathematical language, we can find a solution of the wave equation (13) (we assume we are in the far field and we can consider a plane wavefront) and the resulting decomposition (14) is a linear combination of a basis of orthonormal functions called *spherical harmonics* $Y_{mn}^{\sigma}(\theta,\varphi)$:

$$\nabla^2 p(r,\theta,\varphi,t) - \frac{1}{c^2}\frac{\partial^2 p(r,\theta,\varphi,t)}{\partial t^2} = 0 \qquad (13)$$

where *r* is the radius, $\theta$ is the azimuth and $\varphi$ is the elevation,

$$p(\vec{r}) = \sum_{m=0}^{\infty}(2m+1)\,j^m j_m(kr) \sum_{\substack{0\le n\le m,\\ \sigma=\pm 1}} B_{mn}^{\sigma} Y_{mn}^{\sigma}(\theta,\varphi) \quad (14)$$

where $m$ is the *degree*, $n$ is the *order*, $\sigma$ is the *spin* and $k$ is the *wave number* ($k = 2\pi f/c$). The functions $j_m(kr)$ are called *spherical Bessel functions of the first kind* and the coefficients denoted by $B_{mn}^{\sigma}$ represent the signals recorded by the microphones.

If we represent spherical harmonics in Euler angles, their mathematical formulation is

$$Y_{mn}^{\sigma}(\theta,\varphi) = \sqrt{2m+1}\sqrt{(2-\delta_{0,n})\frac{(m-n)!}{(m+n)!}}P_{mn}\sin\varphi \times \begin{cases} \cos n\theta & \text{if } \sigma=+1 \\ \sin n\theta & \text{if } \sigma=-1 \text{ (ignore } n=0) \end{cases} \qquad (15)$$

where $P_{mn}(\xi)$ is the associated Legendre function of degree *m* and order *n*, $\delta_{pq}$ represents Kronecker delta and it is equal to 1 if $p = q$, else it is equal to 0. The associated Legendre function is defined as

$$\begin{aligned} P_{mn}(\xi) &= \left(1-\xi^2\right)^{\frac{n}{2}}\frac{d^n}{d\xi^n}P_m(\xi) \\ &= \frac{(-1)^m}{2^m m!}\left(1-\xi^2\right)^{\frac{n}{2}}\frac{d^{m+n}}{d\xi^{m+n}}\left(1-\xi^2\right)^m \end{aligned} \qquad (16)$$

where $\xi = \cos\varphi$ .

In the 1st order B-format the omnidirectional microphone is related to the 0th order spherical harmonic ($B_{00}^{1} = B_W$). The three figure-of-eight microphones are related to the 1st order spherical harmonics ($B_{11}^{1} = B_X$, $B_{10}^{1} = B_Y$, $B_{11}^{-1} = B_Z$).

In order to get the B-Format into a single quaternionic signal, we merely have to convert the spherical harmonics from Euler representation into quaternions (Table 1).

The resulting quaternion-valued ambisonic signal has the form

$$B^Q = B_W + B_X\mathbf{i} + B_Y\mathbf{j} + B_Z\mathbf{k}. \qquad (17)$$

**Table 1**. Spherical harmonics up to 1st order: Euler to Quaternion

| Ord. | $m$,$n$,$\sigma$ | Ch. | Euler Spherical Harmonics (normalized) | Quaternion Spherical Harmonics |
|---|---|---|---|---|
| 0 | 0,0,1 | W | 1 | 1 |
| 1 | 1,1,1 | X | $\cos\theta\cos\varphi$ | $x\mathbf{i}$ |
| 1 | 1,0,1 | Z | $\sin\varphi$ | $z\mathbf{k}$ |
| 1 | 1,1,-1 | Y | $\sin\theta\cos\varphi$ | $y\mathbf{j}$ |

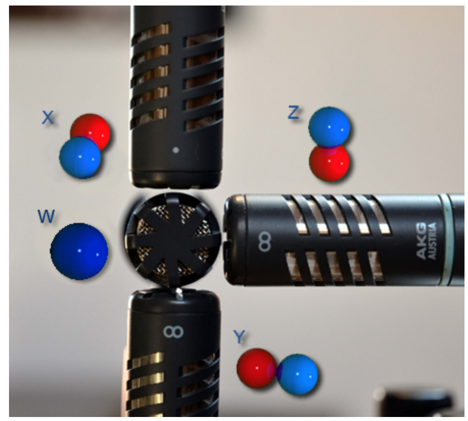


**Fig. 1**. Example of Ambisonic 1st order B-Format layout.

## 5. DIRECT SYSTEM MODELING WITH AMBISONIC SIGNALS

Given a system $W_0(z)$ to be identified, we compare the behavior of the WL-BQLMS and the BQLMS algorithms when the adaptive filter has an improper input signal. The improper quaternion input signal was recorded by means of a 1st order B-Format array as defined in Section 4. The signal samples have the form

$$B^Q[n] = B_W[n] + B_X[n]\mathbf{i} + B_Y[n]\mathbf{j} + B_Z[n]\mathbf{k} \qquad (18)$$

and its source is a monodimensional unit-variance white Gaussian noise. Signal $B_W[n]$ is the omnidirectional component and signals $B_X[n], B_Y[n], B_Z[n]$ are the three figure-of-eight components. The source was placed at a distance of 20 cm from the array, 45° off-axis with the $X$ microphone. The recording environment is an anechoic simulated room. Additive unit-variance white Gaussian noise $\nu[n] \in \mathbb{H}$, with $n = 0, 2..., P-1$, is summed to the output signal of the system to be identified ($d[n] = \mathbf{w}_0^T\mathbf{B}^Q + \nu[n]$). Signal *impropermess* can be easily proved by checking the properties 1 and 2 listed in Section 2.3 for Q-proper signals. As a matter of simplicity, the filter length $M$ in WL-BQLMS is chosen equal to the block length $L$ ($M = L$). Choosing the filter parameters as $M = 4, \mu = 0.6$ ($\mu_B = \mu L$), we obtain the results reported in Fig. 2.

It is worth noting that, when the input signal is improper, non-widely linear algorithms, such as (B)QLMS, typically

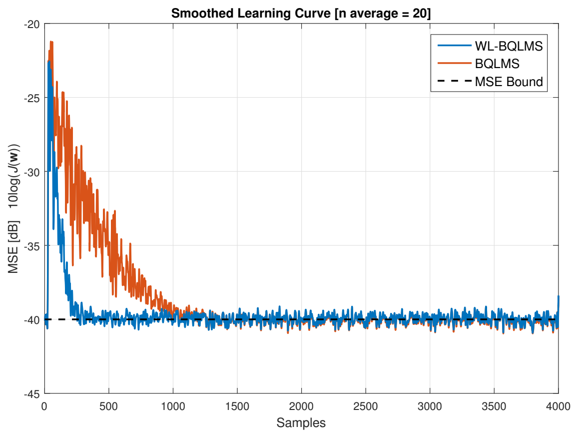


**Fig. 2**. WL-BQLMS vs. BQLMS. Direct system modeling with quaternion-valued ambisonic input signal (improper). Mean Square Error (MSE)

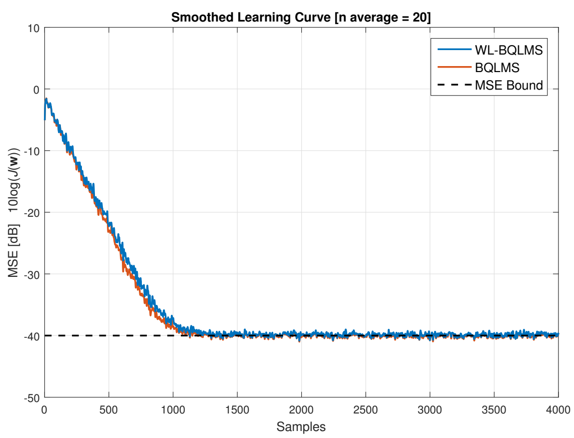


**Fig. 3**. WL-BQLMS vs. BQLMS. Direct system modeling with quaternion-valued proper input signal. Mean Square Error (MSE)

converge slower than widely linear algorithms [19,27]. When the input signal is proper, widely linear and non-widely linear algorithms should not perform differently (aside from minor numerical artefacts). As a proof, we propose another simple experiment of the same kind. This time the quaternion input signal is a proper colored signal defined as $x[n] = bx[n-1] + \frac{\sqrt{1-b^2}}{\sqrt{4}}\eta[n]$, where $\eta[n] \in \mathbb{H}$ with $n = 0, 1, ..., P-1$ is a unit-variance white Gaussian noise sequence and $b$ is a filtering coefficient (chosen as $b = 0.7$ in this simulation). The filter parameters are chosen as $M = 4, \mu = 0.01$. The MSE curve is given in Fig. 3.

## 6. CONCLUSION

Quaternion-valued adaptive filters are known for their property of exploiting the cross-correlation among all components of a multidimensional (quaternion-valued) signal. However, in order to improve the filter performance, it is necessary to design filters capable of handling signals of any nature. We saw that a complete insight of quaternion second order statistics is needed in order to fully exploit the information coupling within all quaternion channels. Widely linear quaternion algorithms were introduced and showed improved performance with the processing of improper signals. However, the computational cost is an issue. We proposed a version of the WL-QLMS algorithm operating weight adaptation periodically (WL-BQLMS). We finally employed the WL-BQLMS algorithm in a typical improper environment such as Ambisonics and compared its performance with its non-widely linear predecessor BQLMS.

## 7. REFERENCES


[1] D. Comminiello, M. Scarpiniti, R. Parisi, and A. Uncini, “Intelligent acoustic interfaces for immersive audio,” in *Proc. 134th Convention of the Audio Engineering Society (AES)*, 2013.

[2] D. Comminiello, S. Cecchi, M. Scarpiniti, M. Gasparini, L. Romoli, F. Piazza, and A. Uncini, “Intelligent acoustic interfaces with multisensor acquisition for immersive reproduction,” *IEEE Trans. Multimedia*, vol. 17, no. 8, pp. 1–11, Aug 2015.

[3] S. D. Chikte and J. Ting-Ho Lo, “Optimal filters for bilinear systems with nilpotent lie algebras,” *IEEE Trans. Automatic Control*, vol. AC-24, no. 6, pp. 948 – 953, 1979.

[4] G. Bourmaud, R. Mégret, A. Giremus, and Y. Berthoumieu, “Discrete extended Kalman filter on Lie groups,” in *Proc. 21st Eur. Signal Process. Conf. (EUSIPCO)*, Sept. 9-13 2013, pp. 1 – 5.

[5] A. Katunin, “Three-dimensional octonion wavelet transform,” *J. Applied Mathematics and Computational Mechanics*, vol. 13, no. 1, pp. 33 – 38, 2014.

[6] E. Hitzer, “Introduction to Clifford’s geometric algebra,” *SICE J. Control, Measurement, and System Integration*, vol. 4, no. 1, pp. 1 – 10, 2011.

[7] C. C. Took and D. P. Mandic, “The quaternion LMS algorithm for adaptive filtering of hypercomplex processes,” *IEEE Trans. Signal Process.*, vol. 57, no. 4, pp. 1316–1327, 2009.

[8] C. Jahanchahi, C. C. Took, and D. P. Mandic, “A class of quaternion-valued affine projection algorithms,” *Signal Process.*, vol. 93, no. 7, pp. 1712 – 1723, Jul. 2013.

[9] C. Jahanehahi, C. C. Took, and D. P. Mandic, “The widely linear quaternion recursive least squares filter,” in *Proc. IEEE 2nd Int. Workshop Cognitive Information Processing (CIP)*, Jun. 14-16 2010, pp. 87 – 92.

[10] T. Nitta, “An extension of the back-propagation algorithm to quaternion,” in *Proc. 3rd Int. Conf. Neural Information Process. (ICONIP)*, Sep. 26 1996, pp. 247 – 250.

[11] A. J. Hanson, *Visualizing Quaternions*, Elsevier, 2006.

[12] B. Roth, *Advances in Robot Kinematics and Computational Geometry*, Springer, 1994.

[13] P. Berner, “Technical concepts - orientation, rotation, velocity and acceleration, and the srm,” "http://www.sedris.org/wg8home/Documents/WG80485.pdf", 2008, [Online].

[14] E. B. Dam and M. M. Koch, “Quaternions, interpolation and animation,” "http://web.mit.edu/2.998/www/QuaternionReport1.pdf", 1998, [Online].

[15] C. C. Took, G. Strbac, K. Aihara, and D. P. Mandic, “Quaternion-valued short-term joint forecasting of three-dimensional wind and atmospheric parameters,” *Renewable Energy*, vol. 36, no. 6, pp. 1754–1760, Jun. 2011.

[16] B. C. Ujang, C. C. Took, and D.P. Mandic, “Quaternion-valued nonlinear adaptive filtering,” *IEEE Trans. Neural Netw.*, vol. 22, no. 8, pp. 1193 – 1206, Aug. 2011.

[17] F. Ortolani and A. Uncini, “A new approach to acoustic beamforming from virtual microphones based on Ambisonics for adaptive noise cancelling,” in *Proc. IEEE 36th Int. Conf. on Electronics and Nanotechnology (ELNANO)*, Apr. 19-21 2016, pp. 337 – 342.

[18] E. Kraft, “A quaternion-based unscented Kalman filter for orientation tracking,” in *Proc. IEEE 6th Int. Conf. of Information Fusion (FUSION)*, Jul. 8-11 2003, vol. 1, pp. 47 – 54.

[19] C.C. Took and D. P. Mandic, “Augmented second-order statistics of quaternion random signals,” *Signal Process.*, vol. 91, no. 2, pp. 214 – 224, Feb. 2011.

[20] F. D. Neeser and J. L. Massey, “Proper complex random processes with applications to information theory,” *IEEE Trans. Inf. Theory*, vol. 39, no. 4, pp. 1293 – 1302, 1993.

[21] N. N. Vakhania, “Random vectors with values in quaternion Hilbert spaces,” *Theory of Probability and its Applications*, vol. 43, no. 1, pp. 18 – 40, 1998.

[22] L. Anttila, M. Valkama, and M. Renfors, “Frequency-selective I/Q mismatch calibration of wideband direct-conversion transmitters,” *IEEE Trans. Circuits Syst. II, Exp. Briefs*, vol. 55, no. 4, pp. 359 – 363, 2008.

[23] P. J. Schreier and L. L. Scharf, *Statistical Signal Processing of Complex-Valued Data: The Theory of Improper and Noncircular Signals*, Cambridge University Press, 2010.

[24] J. Chen and J. Benesty, “Binaural noise reduction in the time domain with a stereo setup,” *IEEE Trans. Audio, Speech, Lang. Process.*, vol. 19, no. 8, pp. 2260–2272, Nov 2011.

[25] J. Cao, A. W. H. Khong, and S. Gannot, “On the performance of widely linear quaternion based MVDR beamformer for an acoustic vector sensor,” in *Proc. 14th Int. Wksp on Acoustic Signal Enhancement (IWAENC)*, 2014, pp. 303 – 307.

[26] T. McWhorter and P. Schreier, “Widely-linear beamforming,” in *Conference Record of the Thirty-Seventh Asilomar Conference on Signals, Systems and Computers (ACSSC)*, Nov. 9 - 12 2003, vol. 1, pp. 753 – 759.

[27] C.C. Took and D. P. Mandic, “A quaternion widely linear adaptive filter,” *IEEE Trans. Signal Process.*, vol. 58, no. 8, pp. 4427 – 4431, 2010.

[28] F. Ortolani and A. Uncini, “Widely linear quaternion adaptive filtering in the frequency domain,” in *Proc. 16th IEEE International Conference on Electromagnetic Theory (MMET)*, 2016.

[29] B. C. Ujang, C. Jahanchahi, C. C. Took, and D. P. Mandic, “Adaptive convex combination approach for the identification of improper quaternion processes,” *IEEE Trans. Neural Netw.*, vol. 25, no. 1, pp. 172 – 181, 2014.

[30] P. O. Amblard and N. Le Bihan, “On properness of quaternion valued random variables,” in *Proc. Int. Conf. on Mathematics (IMA) in Signal Process.*, 2004.

[31] Q. Barthélemy, A. Larue, and J. I. Mars, “About QLMS derivations,” *IEEE Signal Process. Lett.*, vol. 21, no. 2, pp. 240 – 243, Feb. 2014.

[32] F. Rumsey, *Spatial Audio*, Focal Press, 2001.