The Variable Step Size Regularized Block Exact Affine Projection Algorithm

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***Abstract* - This paper presents several block exact affine projection algorithms (BEAPA) with a variable regularization factor and/or variable step size. The performance of the algorithms is investigated for the acoustic echo cancellation (AEC) and noise reduction applications. It is shown that the variable step size regularized BEAPA whose regularization factor and variable step size are adjusted according to the square of a time-averaging estimate of the autocorrelation of *a priori* and *a posteriori* errors is a possible choice for AEC and noise reduction systems.**

Keywords – variable regularization, variable step size, block exact affine projection algorithm, adaptive filter

I. INTRODUCTION

The echo cancellation application is an important system identification problem, where an adaptive filter is used to identify an echo path [1]. The affine projection algorithm (APA) [2] and its fast affine projection (FAP) versions, e.g., [3-5], were found to be very attractive choices for AEC applications. They offer a superior convergence rate as compared to the *normalized least mean square* (NLMS) algorithm, especially for speech signals. The performance of APA is governed by many parameters and it can be improved by varying the step-size [6-8], the regularization factor [9-10], the projection order [12-13] etc. All these parameters have to be chosen based on a compromise between fast convergence rate and good tracking capabilities on the one hand, and low misalignment on the other hand. In [14-15], *nonparametric variable step size* (NVSS) NLMS algorithms were proposed. Recently, variable weight updating versions were proposed for APA [16] and proportionate APA [17]. An interesting alternative to variable step size (VSS) versions is the *variable regularization* (VR) versions of APA [10] or the proportionate APA [11]. The regularization matrix for the VR-APA is obtained by imposing the L2-norm of the *a posteriori* error vector to be equal with the system noise vector [10]. In [18] a fast block exact frequency domain version of the affine projection algorithm was derived, termed Block Exact APA (BEAPA). It was shown that BEAPA has a complexity that is comparable to that of Block Exact FAP (BEFAP) proposed in [19].

In this paper, we propose to combine VSS and VR factors in order to obtain a new nonparametric BEAPA. Both VSS and VR formula are obtained by taking into account the square of the time-averaged estimation of the autocorrelation of *a priori* and *a posteriori* errors. The same approach was used in order to obtain the NVSS-NLMS [15], NVSS-APA [8] and Improved Variable Forgetting Factor RLS (IVFF-RLS) algorithms [20]. It was shown in [8] that the NVSS-APA performance is in general inferior to that of VSS-APA [7]. The advantage of the NVSS scheme of [15] is that it does not use square roots operations that could be difficult to implement on DSPs. We also assume that the filter converges to some extent, thus obtaining new variable regularization formulas. The regularization factor and step size are incorporated in the *block exact APA* (BEAPA) and compared with the VSS-BEAPA proposed in [21].

The paper is organized as follows: Section II describes the variable step size and regularized block exact affine projection algorithms. Simulation results that compare the proposed algorithm with other algorithms are presented in Section III. Finally, the conclusions are presented in Section IV.

II. PROPOSED ALGORITHMS

An unknown system may be identified by using an adaptive filter and both systems have finite impulse responses, defined by the real-valued vectors $\mathbf{h} = [h_0\ h_1\ \ldots\ h_{N-1}]^T$ and $\hat{\mathbf{h}}(n) = [\hat{h}_0(n)\ \hat{h}_1(n)\ \ldots\ \hat{h}_{L-1}(n)]^T$, where superscript T denotes transposition and n is the time index; N is the length of the echo path, while L is the length of the adaptive filter. The signal $x(n)$ is the far-end speech which goes through the acoustic impulse response $\mathbf{h}$, resulting in the echo signal, $y(n)$. This signal is picked up by the microphone together with the noise near-end signal $v(n)$, yielding the microphone signal $d(n)$. The near-end signal can contain both the background noise, $w(n)$, and the near-end speech, $u(n)$.

The APA [2] is defined by the following relations:

$$\mathbf{e}(n) = \mathbf{d}(n) - \mathbf{X}^T(n)\hat{\mathbf{h}}(n-1), \quad (1)$$

$$\hat{\mathbf{h}}(n) = \hat{\mathbf{h}}(n-1) + \mathbf{X}(n)\left[\delta(n)\mathbf{I}_p + \mathbf{X}^T(n)\mathbf{X}(n)\right]^{-1}\mu(n)\mathbf{e}(n), \quad (2)$$

where $\delta(n)$ is the regularization factor, **e**(n) is the a priori error vector **d**(n) = [$d(n)$, $d(n-1)$, …, $d(n-P+1)$]^T is the desired signal vector of length P, with P denoting the projection order and $\mu(n)$ is the step size factor. The matrix **X**(n) = [**x**(n), **x**($n-1$), …, **x**($n-P+1$)] is the input signal matrix, where **x**($n-p$) = [$x(n-p)$, $x(n-p-1)$, …, $x(n-p-L+1)$]^T (with $p = 0,1,\dots, P-1$) are the input signal vectors. The output of the adaptive filter $\hat{y}(n)=\mathbf{x}^T(n)\hat{\mathbf{h}}(n-1)$, provides a replica of the echo, which will be subtracted from the microphone signal. Using the adaptive filter coefficients at time n, the *a posteriori* error vector can be defined as

$$\boldsymbol{\varepsilon}(n)=\mathbf{d}(n)-\mathbf{X}^T(n)\hat{\mathbf{h}}(n). \quad (3)$$

In [21] a VSS scheme that considered the under-modelling situations was adapted to an exact transposition in the frequency domain of a block APA:

$$\mu(n)=\begin{cases}\mu_f, & if\ n\le L\\ \left|1-\dfrac{\sqrt{\left|\hat{\sigma}_d^2(n)-\hat{\sigma}_{\hat{y}}^2(n)\right|}}{\hat{\sigma}_e(n)+\xi}\right|, & \text{otherwise}\end{cases} \quad (4)$$

where ξ is a very small positive number used in order to avoid division by zero, μ_f is the initial step size value, $\hat{\sigma}_d^2(n)$ and , $\hat{\sigma}_d^2(n)$, $\hat{\sigma}_e^2(n)$ computed recursively as follows

$$\hat{\sigma}_d^2(n)=\lambda\hat{\sigma}_d^2(n-1)+(1-\lambda)d^2(n) \quad (5)$$

$$\hat{\sigma}_e^2(n)=\lambda\hat{\sigma}_e^2(n-1)+(1-\lambda)e^2(n) \quad (6)$$

$$\hat{\sigma}_{\hat{y}}^2(n)=\lambda\hat{\sigma}_{\hat{y}}^2(n-1)+(1-\lambda)\hat{y}^2(n) \quad (7)$$

where λ is a weighting factor chosen as $\lambda = 1 - 1/(KL)$, with $K > 1$ and the initial values are $\hat{\sigma}_{\hat{y}}^2(0)=0$, $\hat{\sigma}_e^2(0)=0$ and $\hat{\sigma}_d^2(0)=0$.

The matrix-vector products of VSS-BEAPA [21] were computed using the fast Fourier transform (FFT) as in [19]. The numerical complexity was further reduced by efficient recursive matrix updates [21]. It is known that a gradual adjustment of the regularization factor could improve the convergence speed and steady-state misalignment of the affine projection algorithms [10]. In [7] and [10] it is assumed that the *a posteriori* error is equal with the noise vector and the solution for $\delta(n)$ is obtained such that

$$E\left\{\left\|\boldsymbol{\varepsilon}(n)\right\|^2\right\}=E\left\{\left\|\mathbf{v}(n)\right\|^2\right\} \quad (8)$$

Like in [7], we suppose that $E\left\{y^2(n)\right\}\cong E\left\{\hat{y}^2(n)\right\}$, therefore we have $\hat{\sigma}_v^2(n)\cong\hat{\sigma}_d^2(n)-\hat{\sigma}_{\hat{y}}^2(n)\triangleq\hat{\sigma}^2(n)$.

Therefore the regularization factor is given by

$$\delta(n)=\begin{cases}\dfrac{L\sigma_x^2\hat{\sigma}(n)}{\sigma_e(n)-\hat{\sigma}(n)+\xi}, & if\ \sigma_e(n)\ge\hat{\sigma}\ (n)\\ \delta(n-1) & \text{, otherwise}\end{cases} \quad (9)$$

This formula implies the use of two square root operations and corresponds to the VR-BEAPA algorithm.

The variable step size formulas from [15] and [8] were derived by imposing the equality between the noise variance and the square of the time-averaged estimation of the autocorrelation of the *a priori* and *a posteriori* errors, i.e., the solution for $\mu(n)$ is obtained such that [8]

$$E\left\{\left\|\boldsymbol{\varepsilon}(n)\mathbf{e}(n)\right\|\right\}=E\left\{\left\|\mathbf{v}(n)\right\|^2\right\} \quad (10)$$

and the step size is given by

$$\mu(n)=\begin{cases}\mu_f, & if\ n\le L\\ \left|1-\dfrac{\left|\hat{\sigma}^2(n)\right|}{\hat{\sigma}_e^2(n)+\xi}\right|, & \text{otherwise}\end{cases} \quad (11)$$

Using a similar approach that led to NVSS-APA in [8] another variable regularization formula can be obtained after straightforward mathematical operations as in [10]:

$$\delta(n)=\begin{cases}\dfrac{L\sigma_x^2\left|\hat{\sigma}^2(n)\right|}{\sigma_e^2(n)-\hat{\sigma}^2(n)+\xi}, & if\ \sigma_e(n)\ge\hat{\sigma}(n)\\ \delta(n-1) & \text{, otherwise}\end{cases} \quad (12)$$

The algorithm using the VR formula of (12) is termed NVR-BEAPA. The NVR-BEAPA is square-root free algorithm therefore it is slightly less complex than VR-BEAPA. Intuitively, by comparing (9) and (12), we can expect that most of the time, the regularization factor of NVR-BEAPA is smaller than that of VR-BEAPA due to the square roots operation at the denominator of (10). For the same reason, the dynamic range of $\delta(n)$ in case of VR-BEAPA is in general higher than that of $\delta(n)$ of NVR-BEAPA. It will be shown in the next section (Fig. 1) that the steady-state and tracking performance of both VR-BEAPA and NVR-BEAPA is inferior to that of VSS-BEAPA. However, it is expected that a combination of variable step size and a variable regularization factor could lead to improvements in the steady state and tracking behavior. Therefore, we propose another algorithm, termed *Variable Step Size Regularized BEAPA* (VSSR-

BEAPA) that uses equation (11) for the step size and (12) for the regularization factor. Its complexity is almost the same with that of VSS-BEAPA (the square roots operations are replaced with few multiplications).

III. Simulation Results

The simulations were performed in an AEC and noise reduction in a teleconference context. The far-end signal, $x(n)$, is either a noise or a speech sequence. For AEC simulations an independent white Gaussian noise signal $w(n)$ is added to the echo signal $y(n)$, with different *signal-to-noise ratios* (SNRs). The weighting factor λ is computed using $K = 2$ or $K = 6$ [7], [20]. The initial regularization factor is $\delta = 20\sigma_x^2$, and $\xi = 10^{-4}$. Two echo paths with different sparseness measure were used. The tracking ability of the algorithms is investigated by an abrupt change of the echo path that is introduced by shifting the impulse response to the right by 12 samples. Also, the behavior in case of non-stationary environments is simulated by using a variable background noise level.

In Fig. 1 the performance of VR-BEAPA, NVR-BEAPA, VSS-BEAPA and NVSS-BEAPA are compared and the need for a combination of a variable step size and regularization is justified. A colored input was used as an input, the SNR =30 dB, P = 2, and a change of the echo path is simulated after 1 second. Fig. 1 shows that NVSS-BEAPA has the best tracking performance, but also the worst steady state performance among the considered algorithms and confirms the similar conclusions of [8] regarding NVSS-APA. Both VR versions have slower convergence rate, but can attain a lower steady state misalignment than the VSS versions. Overall, the VSS-BEAPA offers the best performance compromise.

Fig. 2a shows that VSSR-BEAPA has the fastest convergence and lowest steady-state misalignment among the considered algorithms when a speech sequence is applied with a white noise with SNR=20 dB. The projection order is P = 4, and $\mu_f = 1$. Figs. 2b and 2c shows the variation in time of the regularization factor and step-size respectively.

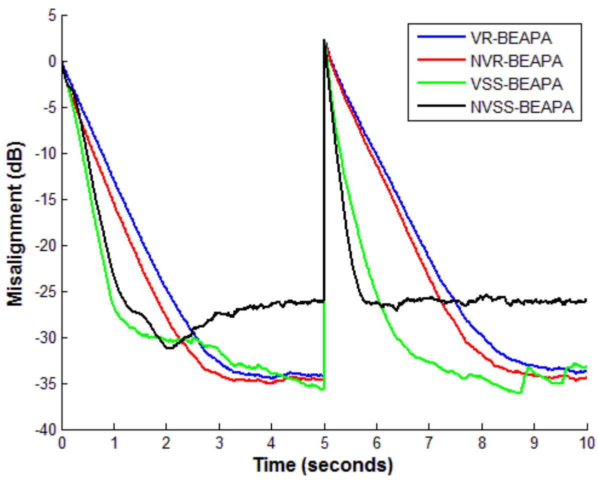

Figure 1. Misalignment of VR-BEAPA, NVR-BEAPA, VSS-BEAPA, NVSS-BEAPA. The input signal is a colored noise sequence, $P = 2$, $L = 512$, echo path changes after 5 seconds, SNR= 30 dB.

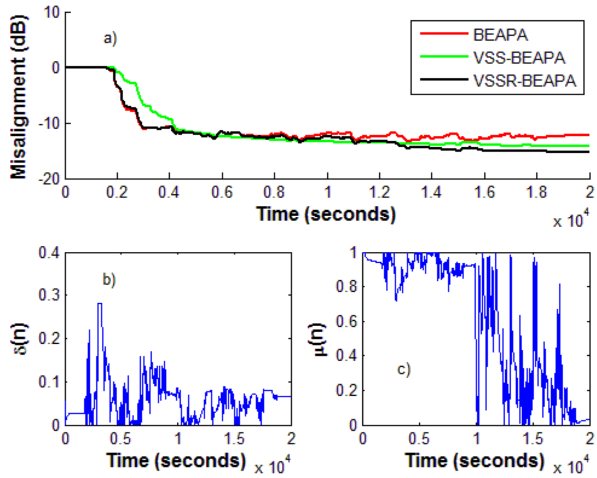

Figure 2. The input signal is a speech sequence, $P = 4$, $L = 300$, SNR= 20 dB. a) Misalignment of BEAPA, VSS-BEAPA, VSSR-BEAPA; b) The variable regularization factor of VSSR-BEAPA; c) The variable step size of VSSR-BEAPA.

The same behavior of the variable parameters has been observed for different projection orders and SNR values.

In Fig. 3, the input is a speech signal, the echo path is sparse and the background noise is variable (SNR decreases from 25 dB to 15 dB between times 1s and 2s, otherwise is 25 dB). The projection order is P = 2, and $\mu_f = 0.2$. It can be seen that VSS-BEAPA attains a lower misalignment than VSSR-BEAPA, but it has the worst tracking ability too.

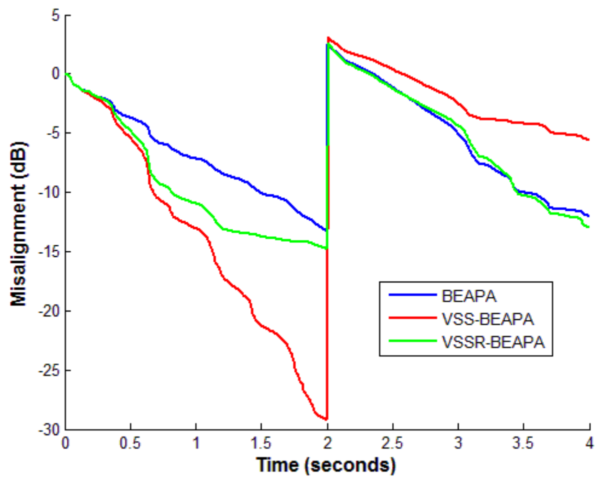

Figure 3. Misalignment of BEAPA, VSS-BEAPA, VSSR-BEAPA. The input signal is a speech sequence, $P = 2$, $L = 512$, and variable background noise (SNR decreases from 25 dB to 15 dB between times 1s and 2s, otherwise is 25 dB).

We also investigate the effectiveness of the VSSR-BEAPA in a noise reduction application in a teleconferencing scenario using a *generalized sidelobe canceller* (GSC) beamformer, as done in [21]. The source of interest is a female speaker located

70 cm from the center of a microphone array. The adopted microphone array is a *uniform linear array* (ULA) composed of 8 omnidirectional microphones placed at a distance of 4 cm each other. The noise source is a diffuse coloured Gaussian noise generated by means of a first-order autoregressive model. Noise reduction results, in terms of signal-to-noise ratio (SNR), are collected in Table I. Although SNR output values for VSSR-BEAPA are close to that of VSS-BEAPA, they confirm the effectiveness of the VSSR-BEAPA even for noise reduction applications. In particular, VSSR-BEAPA outperforms other BEAPA-based algorithms for high noise levels.

Algorithm / *Input SNR*	***BEAPA***	***VSS-BEAPA***	***VSSR-BEAPA***
30 dB	58.2	60.1	60.1
10 dB	58.0	59.9	60.0
0 dB	49.8	54.3	57.7
-5 dB	49.1	52.8	55.1
-10 dB	40.2	48.5	53.4

TABLE I. NOISE REDUCTION PERFORMANCE COMPARISON IN TERMS OF SNR VALUES IN dB.

Future work will be focused on using VSSR-BEAPA in other application domains. Also, the possibility of a simultaneous use of VSS and variable regularization for other AP algorithms will be investigated.

IV. CONCLUSIONS

In this paper, several block exact affine projection algorithms with variable parameters that do not require additional parameters from the acoustic environment were proposed. The simulations proved that the VSSR-BEAPA algorithm could be an alternative to VSS-BEAPA for AEC systems and noise reduction application in a teleconferencing scenario.

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REFERENCES

[1] E. Hänsler and G. Schmidt, Acoustic Echo and Noise Control. A Practical Approach, Hoboken, NJ: JohnWiley & Sons, Inc., 2004

[2] K. Ozeki and T. Umeda, "An adaptive filtering algorithm using an orthogonal projection to an affine subspace and its properties," Electron. Commun. Jpn., vol. 67-A, no. 5, pp. 19–27, May 1984.

[3] S. L. Gay and S. Tavathia, "The fast affine projection algorithm," in Proc. IEEE ICASSP 1995, Detroit, U.S.A.,vol. 5, May 1995, pp. 3023–3026.

[4] A. Gonzalez, M. Ferrer, F. Albu, and M. de Diego, "Affine projection algorithms: evolution to smart and fast multichannel algorithms and applications", in Proc. of Eusipco 2012, Bucarest, Romania, August 2012, pp. 1965-1969.

[5] F. Albu, C. Kotropoulos, "Modified Gauss-Seidel affine projection algorithm for acoustic echo cancellation", in Proc. of ICASSP 2005, Philadelphia, U.S.A., April 2005, pp. 121-124

[6] H.-C. Shin, A. H. Sayed, and W.-J. Song, "Variable step-size NLMS and affine projection algorithms," IEEE Signal Process. Lett., vol. 11, no. 2, pp. 132–135, Feb. 2004.

[7] C. Paleologu, J. Benesty, and S. Ciochina, "A variable step-size affine projection algorithm designed for acoustic echo cancellation," IEEE Trans. Audio, Speech, Lang. Process., vol. 16, pp. 1466-1478, Nov. 2008.

[8] F. Albu, C. Paleologu, and S. Ciochina, "New variable step size affine projection algorithms", in Proc. of COMM 2012, Bucharest, Romania, July 2012, pp. 63-66.

[9] D. Challa, S. L. Grant and M. A. Iqbal, "Variable Regularized Fast Affine Projections," in Proc. of ICASSP 2007, Honolulu, HI, USA, Apr. 2007, pp. 89-92.

[10] W. Yin, and A.S. Mehr, "A variable regularization method for affine projection algorithm", IEEE Trans. Circuits Syst. II: Exp. Briefs, 57, (6), pp. 476–480, 2010.

[11] L. Xiao, Y. Wang, P. Zhang, M. Wu and J. Yang, "Variable regularisation efficient $\mu-$law improved proportionate affine projection algorithm for sparse system identification", IET Elect. Lett., Vol. 48, No. 3, Feb. 2012, pp. 182-184.

[12] S.-E. Kim, S.-J. Kong, and W.-J. Song, "An affine projection algorithm with evolving order," IEEE Signal Process. Lett., vol. 16, no. 11, pp. 937—940, Nov. 2009

[13] F. Albu, C. Paleologu, J. Benesty, "A variable step size evolutionary affine projection algorithm", in Proc. of ICASSP 2011, Prague, Czech Republic, May 2011, pp. 429-432.

[14] J. Benesty, H. Rey, L. Rey Vega, and S. Tressens, "A nonparametric VSS NLMS algorithm," IEEE Signal Process. Lett., vol. 13, no. 10, pp. 581–584, Oct. 2006.

[15] J.-C. Liu and H.-R. Li, "A nonparametric variable step-size NLMS algorithm," Applied Mathematics and Computation, vol. 217, Issue 14, pp. 7365-7371, May 2011.

[16] K.-H. Kim, Y.-S. Choi, S.-E. Kim, and W. -J. Song, "An Affine Projection Algorithm with Periodically Evolved Update Interval", IEEE Trans on Circuits and Systems-II, vol. 58, No. 11, pp. 763-767, 2011.

[17] F. Albu, "New proportionate affine projection algorithm", in Proc. of Inter-Noise/ASME-NCAD 2012, New York, U.S.A., August 2012, 7 pages

[18] G. Rombouts, M. Moonen, "A sparse block exact affine projection algorithm", IEEE Trans. Speech Audio Process., vol. 10, no. 2, pp. 100-108, 2002

[19] M. Tanaka, S. Makino, and j. Kojima, "A block exact fast affine projection algorithm", IEEE Trans. Speech Audio Process., vol. 7, no. 1, pp. 79-86, 1999.

[20] F. Albu, "Improved Variable Forgetting Factor Recursive Least Square Algorithm", in Proc. of ICARCV 2012, December 2012, Guangzhou, China, to appear.

[21] D. Comminiello, M. Scarpiniti, R. Parisi and A. Uncini, "A novel affine projection algorithm for superdirective microphone array beamforming", in Proc. of ISCAS 2010, Paris, France, pp. 2127-2130, 2010.