COMPARISON OF HAMMERSTEIN AND WIENER SYSTEMS FOR NONLINEAR ACOUSTIC ECHO CANCELERS IN REVERBERANT ENVIRONMENTS

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ABSTRACT

The aim of this paper is the presentation of a comparative analysis of Hammerstein and Wiener systems used for the problem of compensation of the nonlinear distortion due to non-ideality of amplifiers and loudspeakers in acoustic echo cancellation. The proposed solutions consist in a cascade of a flexible nonlinear function, whose shape can be modified during the learning process, and a linear adaptive filter in different order, respectively. Two different type of flexible nonlinearities are tested. Some numerical results show the effectiveness of the proposed approaches and underline that the Hammerstein system has better performances than the Wiener one.

***Index Terms*—** Acoustic Echo Cancellation (AEC), Adaptive Filters, Nonlinear Compensation, Hammerstein System, Wiener System.

1. INTRODUCTION

In last years an increasing interest in the problem of acoustic echo cancellation (AEC) has arisen, due to emerging teleconferencing and hands-free telephone systems [1]. Unfortunately the well-known linear algorithms proposed in literature are too unrealistic in many practical situations since their performance is limited by the presence of nonlinearities, like those typically generated in low-quality loudspeakers [2, 3].

Recently some works in nonlinear environment were proposed [4, 5, 6, 7, 8, 9]. The nonlinear functions proposed in these works are inefficient in some problems [3], due to the piecewise nature of nonlinear transformation. In particular the authors in [4] proposed the use of a raised-cosine function, able to adapt both soft-clipping and hard-clipping nonlinearities, while in [5] the authors proposed a two-parameter sigmoid function whose slope and amplitude can be updated during the learning process. The latter was extended in [7] evaluating the echo suppressing performances at different reverberation levels, but both works are limited to the Hammerstein model, i.e. the nonlinearity is preceding the linear filtering. Some of the authors of this paper have already proposed a flexible solution to the problem in [10], but the proposed approach is limited to only Wiener systems.

A more accurate solution is the adoption of truncated Volterra filters, usually applied to nonlinear echo cancellation, but the improvements obtained with respect to linear adaptive filters do not justify a huge computational load [6, 9, 11]. A practical solution consists in the adoption of cascade filters [12]. Unfortunately the work proposed in [12] is limited to suppression of the echo between loudspeaker and microphone in a mobile phone, and no evaluation is conducted in a reverberant environment.

In this paper we propose a comparative analysis of Hammerstein and Wiener systems, architectures composed of a cascade of a flexible adaptive nonlinear function followed by a traditional linear adaptive filter [12, 13] or vice versa, in different reverberant environments. In order to adapt the nonlinear function we propose the use of two nonlinearities: a parametric sigmoid function [5] and spline functions that are smooth parametric curves defined by interpolation of properly defined control points collected in a lookup table [14].

The adaptation of the nonlinearities involved in the architectures and the adaptive filters is performed by a standard LMS algorithm [13].

This paper is organized as follows: section 2 describes the echo canceler architectures and the proposed nonlinearities, while section 3 is devoted to the derivation of the learning rules. Section 4 shows some experimental results demonstrating the effectiveness of the proposed approach in comparison with the existing linear ones. Finally section 5 draws our conclusions.

2. THE NONLINEAR ECHO CANCELER

An acoustic echo canceler is an adaptive system that aims at reducing the echo caused by a sound from a loudspeaker that can be picked up by the microphone in the same room. In addition the difficulties in canceling acoustic echo stem from the alteration of the original sound by the ambient space [13]. This colors the sound that re-enters the microphone. In addition some nonlinear distortions in signals occur due to the low-cost audio equipment.

The effect of the distortions of amplifier and loudspeakers is modeled by a nonlinear function (NL) applied on the input signal. The effect of the environment is modeled by the *Room Impulse Response* (RIR) $h(n)$. This model of a nonlinear AEC (NAEC) is know in literature as Hammerstein system and is shown in Figure 1-a). An alternative model of the nonlinear distortions of amplifier and loudspeakers is modeled by a cascade of the RIR $h(n)$ and a nonlinear function. This latter model of a NAEC is known as Wiener system and is shown in Figure 1-c).

The proposed architecture consists in a cascade of a nonlinear function, implemented by a parametric sigmoid or a spline function and a FIR filter. The system is depicted in Figure 2, where $x(n)$ is the excitation signal, $d(n)$ is the reference signal and $b(n)$ is a local background noise. In the proposed scheme, $u(n) = f(x(n))$ is the distorted signal, $y(n) = \mathbf{w}^T\mathbf{u}$ is the output of the adaptive filter, that is the estimate of distorted echo signal, where $\mathbf{w} = [w_1, w_2, \ldots, w_L]$ are the L coefficients of the adaptive filter and $\mathbf{u} = [u(n), u(n-1), \ldots, u(n-L+1)]$, while $e(n) = d(n) - y(n)$

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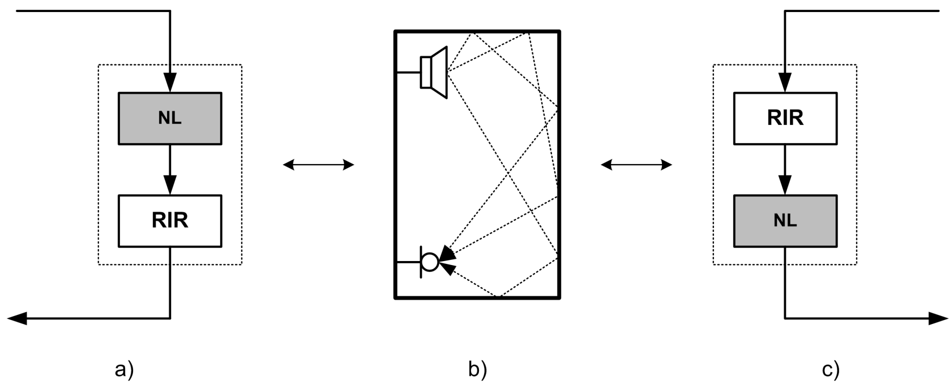

Fig. 1. Two different implementations of a distorting echo path (b): (a) cascade of a nonlinear function (NL) and a Room Impulse Response (RIR) (Hammerstein system) and (c) cascade of a RIR and a NL (Wiener system).

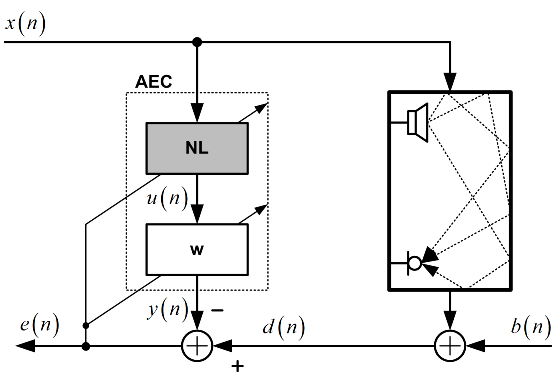

Fig. 2. The proposed Hammerstein NAEC architecture.

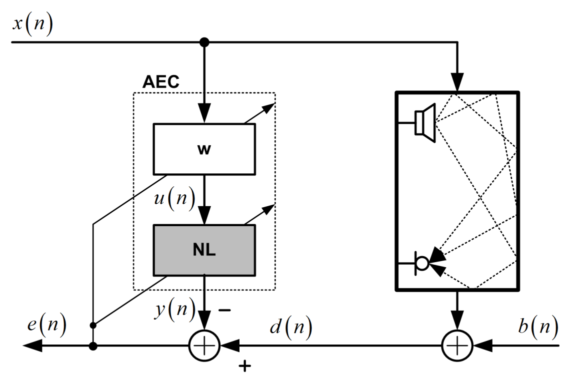

Fig. 3. The proposed Wiener NAEC architecture.

is the error signal. As shown in Figure 2 the architecture consists in the cascade of the nonlinear function and the linear adaptive filter, whose adaptation rules are described in the following section.

Alternatively Figure 3 shows the Wiener counterpart. In this new scheme, $u(n) = \mathbf{w}^T\mathbf{x}$ is the output of the linear adaptive filter, while $y(n) = f(u(n))$ is the distorted output signal. $\mathbf{w} = [w_1, w_2, \ldots, w_L]$ are the L coefficients of the adaptive filter and $\mathbf{x} = [x(n), x(n-1), \ldots, x(n-L+1)]$, while $e(n) = d(n) - y(n)$ is the error signal. In addition an additive environmental noise $b(n)$ can be considered.

2.1. The Nonlinear Transformations

The first idea to implement a flexible nonlinear function is the adoption of a parametric sigmoid function, whose shape can be controlled by two parameters α and β. The mathematical expression is the following one:

$$f(x) = \frac{2\beta}{1 + \exp(-\alpha x)} - \beta. \quad (1)$$

With these two parameters we can control the amplitude and the slope of the sigmoid. In addition this nonlinearity is able to profile a wide class of nonlinearities involved in this kind of nonlinear distortion [5].

The implementation of the flexible function $f(x)$ can be also reached by a spline interpolation scheme [14]. Splines are smooth parametric curves defined by interpolation of properly defined control points collected in a look-up table. In the general case, given N equispaced control points, the spline curve results as a polynomial interpolation through $N-3$ adjacent spans. Let $u = f(x)$ be some function to be estimated. The spline estimation provides an approximation $f(x) \cong \hat{f}(\nu(x), i(x))$ based on two local parameters (ν, i) directly depending on the input x:

$$\begin{aligned} z' &= \tfrac{x}{\Delta} + \tfrac{N-2}{2} \\ z &= \begin{cases} 1, & \text{if } z' < 1, \\ z', & \text{if } 1 \le z' \le N-3, \\ N-3, & \text{if } z' > N-3 \end{cases} \\ i &= \lfloor z \rfloor \\ \nu &= z - i \end{aligned} \quad (2)$$

where z, z' are two internal variables, Δ is the distance between two consecutive control points q_i and q_{i+1} and $\lfloor \bullet \rfloor$ indicates the floor operator that returns the highest integer less than or equal to its input. In this specific application, for each input occurrence $\bar{x}$ the spline estimates $f(\bar{x})$ by using four control points selected inside the look-up table [14]. Two points are the adjacent control points on the left side of $\bar{x}$, while the other two points are the two control points on the right side.

Hence the output of a generic input $\bar{x}$ is simply obtained by the following matrix expression, as explained in detail in [14]:

$$\bar{u} = f(\bar{x}) = \hat{f}(\nu(\bar{x}), i(\bar{x})) = \mathbf{TMQ}_i, \quad (3)$$

where $\mathbf{T} = \begin{bmatrix} \nu^3 & \nu^2 & \nu & 1 \end{bmatrix}$ is a vector depending only by the local abscissa ν, $\mathbf{Q}_i = \begin{bmatrix} q_i & q_{i+1} & q_{i+2} & q_{i+3} \end{bmatrix}^T$ is the vector that collects the local control points and $\mathbf{M}$ is a 4×4 matrix which selects which spline base is used, typically B-spline or Catmull-Rom spline (CR-Spline) [14].

To ensure the monotonously increasing characteristic of the overall function, the additional constraint $q_i < q_{i+1}$ must be imposed.

3. PROPOSED ARCHITECTURES

The adaptation of the proposed architectures (linear filter and parameters of the nonlinear functions) are drawn following the well known Least Mean Square (LMS) algorithm [13]. Thus the cost function adopted is

$$J(n) = |e(n)|^2 = |d(n) - y(n)|^2 . \tag{4}$$

3.1. Hammerstein system

In this way, following the scheme in Figure 2, the learning rule for the adaptation of the filter weights $\mathbf{w}$ at instant n is [13]

$$\Delta \mathbf{w}_n = \frac{\partial J(n)}{\partial \mathbf{w}} = -2e(n)\mathbf{u}_n. \tag{5}$$

Absorbing the term -2 into the learning rate μ_w, the final learning rule becomes

$$\mathbf{w}_{n+1} = \mathbf{w}_n + \mu_w e(n)\mathbf{u}_n. \tag{6}$$

In the case of using the sigmoid function (1), the parameters α and β at instant n are adapted consequently:

$$\Delta \alpha_n = \frac{\partial J(n)}{\partial a} = -2e(n)\mathbf{w}^T \cdot \frac{\partial \mathbf{u}}{\partial \alpha}, \tag{7}$$

for α coefficient and

$$\Delta \beta_n = \frac{\partial J(n)}{\partial b} = -2e(n)\mathbf{w}^T \cdot \frac{\partial \mathbf{u}}{\partial \beta}, \tag{8}$$

for β coefficient. Hence the final updating equations, absorbing the term -2 into the learning rates μ_α and μ_β respectively, are

$$\begin{array}{l} \alpha_{n+1} = \alpha_n + \mu_\alpha e(n)\mathbf{w}^T \cdot \frac{\partial \mathbf{u}}{\partial \alpha}, \\ \beta_{n+1} = \beta_n + \mu_\beta e(n)\mathbf{w}^T \cdot \frac{\partial \mathbf{u}}{\partial \beta}. \end{array} \tag{9}$$

The terms $\partial \mathbf{u}/\partial \alpha$ and $\partial \mathbf{u}/\partial \beta$ depend on which nonlinearity is considered and their expressions are very simple for the nonlinear function in eq. (1).

In a similar way, in the case of adopting the spline function defined in (3), the m-th control point of the spline function at instant n can be adapted

$$\Delta q_m^n = \frac{\partial J(n)}{\partial q_m} = 2e(n)\frac{\partial e(n)}{\partial q_m} = -2e(n)\sum_{k=1}^{L} w_k \frac{\partial u(k)}{\partial q_m}, \tag{10}$$

where $m = 1 \dots 4$. It is easy to show that

$$\frac{\partial u(k)}{\partial q_m} = \mathbf{T}\mathbf{M}_m, \tag{11}$$

where $\mathbf{M}_m$ is the m-th column of the matrix $\mathbf{M}$. Hence the final updating equation, absorbing the term -2 into the learning rate μ_q, is

$$q_m^{n+1} = q_m^n + \mu_q e(n)\sum_{k=1}^{L} w_k \frac{\partial u(k)}{\partial q_m}. \tag{12}$$

3.2. Wiener system

Reasoning in a similar way and following the scheme in Figure 3, we can find similar updating equations in the case of Wiener system.

The learning rule for the adaptation of the filter weights $\mathbf{w}$ at instant n is [13]

$$\mathbf{w}_{n+1} = \mathbf{w}_n + \eta_w e(n)\frac{\partial \mathbf{y}}{\partial \mathbf{u}}\mathbf{x}_n, \tag{13}$$

where $\partial \mathbf{y}/\partial \mathbf{u}$ depends by which nonlinearity is used. In particular, for the spline function (3) it becomes

$$\frac{\partial \mathbf{y}}{\partial \mathbf{u}} = \frac{\partial \mathbf{y}}{\partial \nu}\frac{\partial \nu}{\partial \mathbf{u}} = \frac{1}{\Delta}\dot{\mathbf{T}}\mathbf{M}\mathbf{Q}_i, \tag{14}$$

where $\dot{\mathbf{T}} = \begin{bmatrix} 3\nu^2 & 2\nu & 1 & 0 \end{bmatrix}$. The parameters of the sigmoid function are adapted consequently

$$\begin{array}{l} \alpha_{n+1} = \alpha_n + \eta_\alpha e(n)\mathbf{w}^T \cdot \frac{\partial \mathbf{x}}{\partial \alpha}, \\ \beta_{n+1} = \beta_n + \eta_\beta e(n)\mathbf{w}^T \cdot \frac{\partial \mathbf{x}}{\partial \beta.} \end{array} \tag{15}$$

In the case of adopting the spline function defined in (3) the m-th control point of the spline function at instant n can be adapted by

$$q_m^{n+1} = q_m^n + \eta_q e(n)\mathbf{T}\mathbf{M}_m, \tag{16}$$

where $\mathbf{M}_m$ is the m-th column of the matrix $\mathbf{M}$ and $m = 1 \dots 4$.

4. EXPERIMENTAL RESULTS

This section shows some experimental results demonstrating the effectiveness of the proposed approach. Obtained results are compared with a standard linear echo canceler using an LMS algorithm [1, 13].

Performance is evaluated in terms of *Echo Return Loss Enhancement* (ERLE) [15], which is defined as:

$$ERLE = 10\log_{10}\frac{E\left\{d^2[n]\right\}}{E\left\{e^2[n]\right\}}, \tag{17}$$

where the operator $E\{\cdot\}$ is the mathematical expectation.

The experimental tests were conducted in a simulated environment with different reverberation time T_{60}. The impulse responses of this environment where evaluated using the Matlab toolbox Roomsim[1] in a room of dimensions $6 \times 4 \times 2.5$ m and changing the wall absorption coefficients. The receiver is placed in position $[1.56,\ 1.88,\ 1.1]$ m, while the source is located at 1 m of distance in front of the microphone along the x-direction.

In a first experiment we propose an application of the proposed echo canceler using a white Gaussian noise with unitary variance. The signal has a length of 50000 samples. The nonlinear distortion adopted is a sigmoid function with $\alpha = 4$ and $\beta = 1$. The learning rates are heuristically set to the following values: $\mu_w = \mu_\alpha = \mu_\beta = 10^{-3}$, $\eta_w = \eta_\alpha = \eta_\beta = 10^{-2}$, while $\mu_l = 10^{-4}$ and $\mu_q = \eta_q = 10^{-1}$. B-spline basis is used, the control points are equispaced of $\Delta = 0.1$ and their number is set to 21. The filter coefficients $\mathbf{w} = [w_0, w1, \dots, w_{L-1}]^T$ are initialized to all zeros. The filter length L depends on the reverberation time used, and it is listed in Table 1. As an example Figure 4 shows the ERLE comparison for the case of an anechoic environment. The figure demonstrates that

[1]Roomsim is a MATLAB simulation of shoe-box room acoustics for use in teaching and research. Roomsim is available from `http://media.paisley.ac.uk/~campbell/Roomsim/`.

T_{60} [ms]	L
Anechoic	256
50	512
100	1024
200	1280
350	2048

Table 1. Reverberation time T_{60} and filter length L used in experiments.

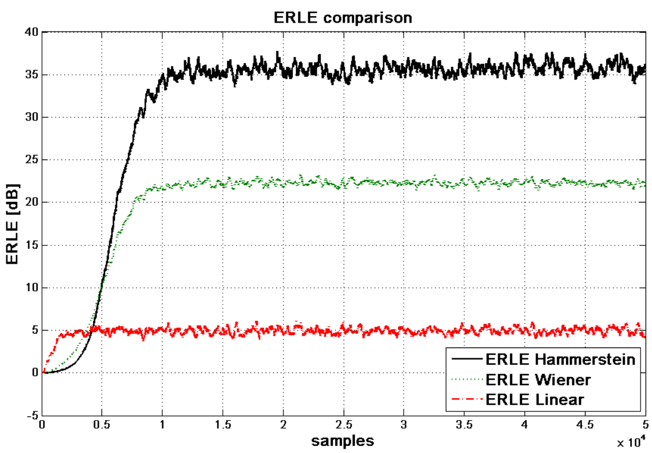

Fig. 4. ERLE comparison for the anechoic environment using sigmoid function.

the proposed approaches outperform the linear AEC and in particular shows that the Hammerstein system has better performances than the Wiener one in terms of ERLE index. In Figure 5 it is shown the ERLE comparison for the case of adopting a spline function in an anechoic environment. It is evident from this figure that the ERLE is lower than the previous case and the convergence speed is quite slower. This can be explained by remembering that in this latter case we must adapt 21 parameters instead of 2 parameters of the sigmoid function. Figure 6 depicts the error signal in the case of Hammerstein system using a sigmoid compensating nonlinear function. In addition Figure 7 depicts the profile of the spline compensating nonlinear function at the end of convergence. As we can see, the approach is able to recover the original distorting function.

Results for white Gaussian noise in all other reverberant environments are summarized in Table 2. These results are averaged over 100 trials. The performance of the Hammerstein system remains good until a $T_{60} = 200$ ms, then degrades gradually at $T_{60} = 350$ ms, although it is appreciable. The Wiener system provides poorer performance. Worst is the case of using a spline function, but this nonlinearity is able to adapt any kind of distortion unlike the sigmoid one for which the performances dramatically fall down. In any case the performances of the proposed architectures highly outperform the ERLE of the linear system, too simple and unrealistic.

The same experimental tests are conducted using 30000 samples of a male speech signal sampled at 8000 Hz. We use the same parameters of the previous example. In addition a background noise $b(n)$ with $SNR = 20$ dB is added to the input signal. Results, averaged over 100 trials, at different reverberation time are summarized in Table 3. The trends are the same as the previous case, but the upper bound is lower due to the non-stationary nature of speech

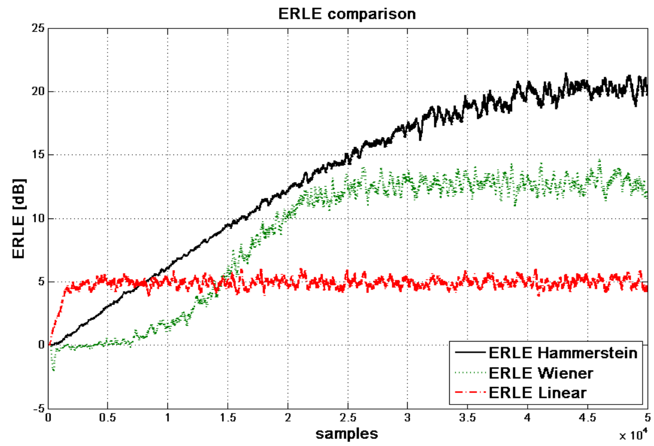

Fig. 5. ERLE comparison for the anechoic environment using spline function.

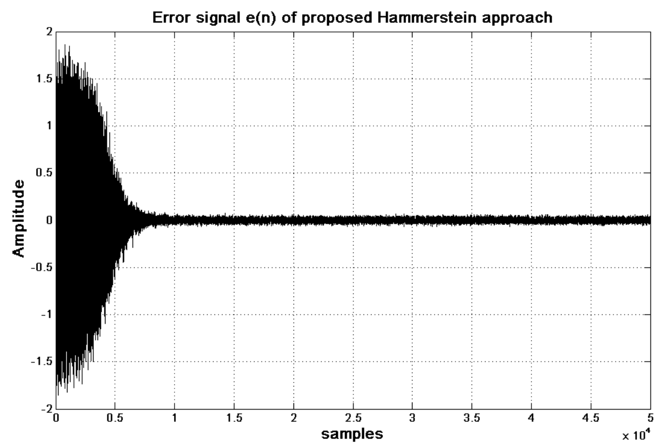

Fig. 6. Error signal of the Hammerstein system with sigmoid function in an anechoic environment.

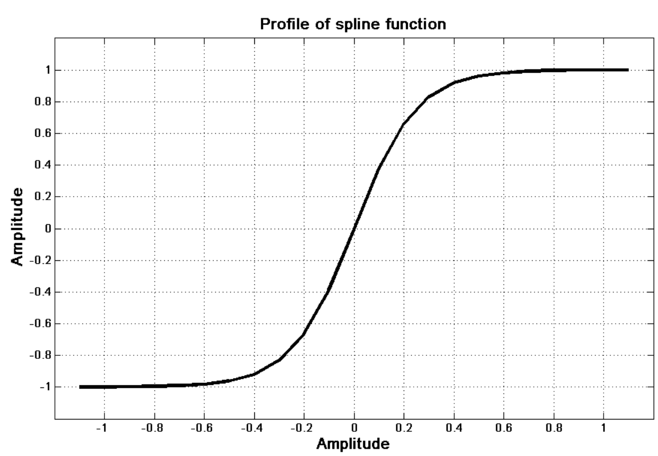

Fig. 7. Profile of the estimated loudspeaker nonlinearity using a spline function for the anechoic environment.

NL	System	Anechoic	50 [ms]	100 [ms]	200 [ms]	350 [ms]
Sigmoid	Hammerstein	35.07	33.68	31.03	24.67	12.63
	Wiener	22.79	21.46	18.83	15.02	7.75
Spline	Hammerstein	23.38	21.62	19.72	16.80	11.56
	Wiener	19.38	18.62	17.72	14.82	9.56
Linear		5.01	4.95	4.94	4.55	4.14

Table 2. Summary of ERLE [dB] for the proposed experimental tests with white Gaussian noise signal at different T_{60}.

NL	System	Anechoic	50 [ms]	100 [ms]	200 [ms]	350 [ms]
Sigmoid	Hammerstein	21.39	18.40	16.42	13.88	9.22
	Wiener	19.27	17.26	14.11	11.21	6.37
Spline	Hammerstein	20.49	17.93	16.93	14.31	9.59
	Wiener	19.26	15.66	13.61	21.40	6.44
Linear		3.81	3.75	3.53	2.65	2.30

Table 3. Summary of ERLE [dB] for the proposed experimental tests with a male speech signal at different T_{60}.

signal.

Moreover we evaluated the *Misalignment* between the room impulse response $\mathbf{h}$ and the estimated one $\mathbf{w}_n$ at the instant n, defined by [16, p. 78]

$$D(n) = 10 \log_{10} \frac{\|\mathbf{h} - \mathbf{w_n}\|_2^2}{\|\mathbf{h}\|_2^2}. \qquad (18)$$

The values of the misalignment (18), averaged over 100 trials, are summarized in Table 4 for the white Gaussian noise signal. These results confirm those proposed in Table 2: the Hammerstein system is able to identify well the RIR even in a high reverberant environment, while the Wiener system produce poorer misalignments. As above worst results are obtained using a spline function.

Table 5 summarizes the misalignment (18) in the case of the male speech signal. This table shows that the misalignment is poorer but it is quite stable while varying the reverberation level.

Furthermore from a computational complexity point of view, the algorithms based on the sigmoid function and the spline function are comparable. In fact the adaptation of the sigmoid function involves two parameters, while the adaptation of the spline function involves four parameters and does not change the order of algorithm complexity.

5. CONCLUSION

In this paper we have introduced a comparative analysis of Hammerstein and Wiener systems for a nonlinear acoustic echo canceler. The nonlinear functions involved in the architectures are implemented using parametric sigmoid functions and spline functions, whose shape can be changed during the learning process. The comparisons are conducted at different reverberation levels, measured in terms of reverberation time or T_{60}. The performances of the proposed schemes are measured in terms of ERLE index and Misalignment, well known in literature.

The experimental results have demonstrated that Hammerstein system provides better performances than Wiener system. Moreover, even if the spline nonlinearity is able to compensate any kind of distorting function, the simpler flexible sigmoid nonlinearity provides better results in compensating the simple distortion used in this work.

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NL	System	Anechoic	50 [ms]	100 [ms]	200 [ms]	350 [ms]
Sigmoid	Hammerstein	-40.65	-36.16	-33.21	-31.82	-26.41
	Wiener	-29.47	-25.89	-21.96	-17.92	-10.02
Spline	Hammerstein	-23.84	-22.87	-20.25	-15.00	-10.27
	Wiener	-14.99	-12.70	-10.60	-9.04	-5.21

Table 4. Summary of misalignment $D(n)$ [dB] for the proposed experimental tests with white Gaussian noise signal at different T_{60}.

NL	System	Anechoic	50 [ms]	100 [ms]	200 [ms]	350 [ms]
Sigmoid	Hammerstein	-14.44	-12.38	-11.37	-11.12	-9.25
	Wiener	-12.12	-11.03	-10.95	-9.92	-9.02
Spline	Hammerstein	-13.51	-12.11	-9.26	-8.81	-6.24
	Wiener	-10.03	-8.70	-8.17	-6.25	-4.93

Table 5. Summary of misalignment $D(n)$ [dB] for the proposed experimental tests with a male speech signal at different T_{60}.

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